

Assignment 1

HMTH407/HFM307 (Partial Differential Equations)

September 2022

1. For each of the following equations, state the order and whether it is nonlinear, linear inhomogeneous or linear homogeneous.

(a) (2 marks)

$$(\cos xy^2)u_x - y^2 u_{xxy} = \tan(x^2 + y^2)$$

(b) (2 marks)

$$\frac{u_x}{\sqrt{1+u_x^2}} + \frac{u_y}{\sqrt{1+u_y^2}} = 0.$$

2. (6 marks) Solve the equation

$$(1+x^2)u_x + u_y = 0.$$

Sketch some of the characteristic curves.

3. (5 marks) Solve the equation

$$\sqrt{1-x^2}u_x + u_y = 0, \quad u(0, y) = y$$

4. (5 marks) Solve

$$u_x + u_y + u = e^{x+y} \quad u(x, 0) = 0.$$

5. (15 marks) Consider the one-dimensional wave equation

$$u_{tt} - c^2 u_{xx} = 0, \quad c > 0$$

- (a) Show that the wave equation is hyperbolic.
(b) By using the method of characteristics, deduce that the general formula for the wave equation is given by

$$u(x, t) = F(x + ct) + G(x - ct),$$

where F, G are arbitrary.

- (c) The wave equation is supplemented with the initial conditions

$$u(x, 0) = f(x), \quad u_t(x, 0) = g(x).$$

Deduce d'Alembert's formula:

$$u(x, t) = \frac{1}{2}[f(x+ct) + f(x-ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} g(s) ds.$$

- (d) Show that the d'Alembert formula above satisfies the wave equation.
(You may assume that $f''(x)$ and $g'(x)$ exist and are continuous.)
- (e) Solve the wave equation

$$\begin{aligned} u_{tt} - 25u_{xx} &= 0, \\ u(x, 0) &= e^{-x^2}, \\ u_t(x, 0) &= 4 + x. \end{aligned}$$

6. (15 marks) Consider the equation $u_{xx} - 2u_{xy} + u_{yy} + u_x + u_y = 0$.

- (a) What is its type (elliptic/parabolic/hyperbolic)?
- (b) Reduce it to standard form.
- (c) Find its general solution.