Assignment 2

HMTH407/HFM307 (Partial Differential Equations)

September 27, 2022

1. (10 marks) Solve the Cauchy problem for the heat equation

$$u_t - ku_{xx} = 0, \quad -\infty < x < \infty, \quad t > 0$$
$$u(x,0) = \phi(x),$$

where

$$\phi(x) = \begin{cases} 5, & \text{if } |x| < 2, \\ 0, & \text{if } |x| > 2. \end{cases}$$

leaving your solution in terms of the error function.

2. (10 points) Use Fourier Transforms to solve the Cauchy problem for the heat equation

$$u_t - ku_{xx} = 0, \quad -\infty < x < \infty, \quad t > 0.$$
$$u(x,0) = \phi(x).$$

3. (10 points) Use Fourier Transforms to show that the solution of the Dirichlet problem for the Laplace equation in the upper-half plane

$$u_{xx} + u_{yy} = 0, \quad x \in \mathbb{R}, \quad y > 0,$$
$$u(x,0) = g(x).$$

is given by

$$u(x,y) = \frac{y}{\pi} \int_{-\infty}^{\infty} \frac{f(\tau)}{(x-\tau)^2 + y^2} \, d\tau.$$

4. (10 points) Use the result from Question 4 to show that the solution of the Dirichlet problem for the Neumann problem for the Laplace equation

$$u_{xx} + u_{yy} = 0, \quad x \in \mathbb{R}, \quad y > 0,$$
$$u_y(x,0) = f(x).$$

is given by

$$u(x,y) = \int_{-\infty}^{\infty} g(x-\xi) \left[\ln(y^2 + \xi^2) - \ln(\xi^2)\right] \, d\xi$$

Hint: Use the transformation $v = u_y$ and reduce the problem to a Dirichlet problem.

5. (10 points) Logan 4.1: 1.