HMTHCS 212 Assignment 2 Name _____

This assignment is due on **Wednesday July 13, 2022** by 6pm.

1. (30 points)

   (a) Use Gaussian elimination solve the following system of equations, if possible, and determine if row interchanges are necessary.

$$\begin{aligned} x_1 + x_2 + x_4 &= 2 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \\ -x_1 + 2x_2 + 3x_3 - x_4 &= 4 \\ 3x_1 - x_2 - x_3 + 2x_4 &= -3 \end{aligned}$$

   (b) Use Gaussian elimination with backward substitution to solve the following linear system. Do not re-order the equations.

$$\begin{aligned} 4x_1 + x_2 + 2x_3 &= 9 \\ 2x_1 + 4x_2 - x_3 &= -5 \\ x_1 + x_2 - 3x_3 &= -9 \end{aligned}$$

   (c) Show that the LU factorization of the matrix A from part (b) is $A = LU$, where:

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & \frac{3}{14} & 1 \end{pmatrix}, \text{ and } U = \begin{pmatrix} 4 & 1 & 2 \\ 0 & \frac{7}{2} & -2 \\ 0 & 0 & -\frac{43}{14} \end{pmatrix}$$

   (d) Use the LU factorization from part (c) to solve the linear system in part (b).

   (e) What is a diagonally dominant matrix? Show that the matrix A from part (b) is diagonally dominant.

   (f) Perform 3 steps of the Jacobi iteration to approximate the solution of the linear system from part (b).

   (g) Perform 3 steps of the Gauss-Seidel iteration to approximate the solution of the linear system from part (b).

2. (20 points) Consider the following table of data. Find a polynomial of degree 3 that

x	-1	0	1	2
y	3	5	5	27

interpolates this data using the following methods. Show that the polynomial is the same in all three cases.

   (a) monomial interpolation

   (b) Lagrange's interpolation

   (c) Newton's interpolation (using a divided differences table)

3. (10 points) A quadratic polynomial $p(x)$ is used to approximate the function $f(x) = e^x$ on the interval $[-1, 1]$. The interpolating polynomial passes through the points $x = -1, 0, 1$

   (a) Find the interpolating polynomial $p(x)$.

   (b) Write down the expression for the interpolation error $E(x) = |e^x - p(x)|$

   (c) Hence, find the maximum possible value of the interpolation error $E(x)$ on the interval $[-1, 1]$.