USING OPERATIONS RESEARCH ANALYTIC TECHNIQUES TO SOLVE THE LAND ALLOCATION, TRANSPORTATION PROBLEM AND DECISION ANALYSIS TO SMALL-SCALE FARMING: CASE TWO FARMS IN NORTON AND JOETECH PRIVATE COMPANY

by

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To my family and friends

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Abstract

With the increase in the cost of resources in Zimbabwe, farmers have to maximize their returns from the scarce resources. At the same time there has been a rapid increase in the country's populations yet resources are limited to satisfy food demand. With the development and use of analytic techniques in operations research, more farm management problems can be reduced and enable farmers to be successful and feed the nation. Since small-scale farming is growing rapid in Zimbabwe, there is need for small scale farmers to be equipped with important knowledge on how to advance their farm management using these advanced mathematical and statistical techniques.

This paper discuses four important problems that small scale farmers have to solve in order to maximize their returns. The problems are the land allocation problem, maximizing revenue from sale of livestock, transportation problem in agriculture and making decisions under risk and uncertainty. This paper solves the land allocation problem, maximizes revenue from sale of livestock and the transportation problem for a farm owner who owns two farms in Norton and currently uses traditional methods of farming. The paper further helps farmers on how to a make decision under risk and uncertainty using the case of JoeTech Private company that wants to venture into farming at a small scale. For the land allocation problem, maximizing revenue from selling livestock and transportation problem, data was collected and used to formulate linear programming models to solve these problems.

When the land allocation problem was solved, optimal allocations of land was obtained that would maximize the farmer's revenue. It was found that the farmer should allocate more land to tobacco, wheat and sunflower. Land would be allocated 50 ha each and the farmer would be guaranteed to maximize revenue if these crops are sold. The transportation problem also produced results that showed the best allocations of goats to desired destinations at the least transportation cost. The transportation problem showed that the farmer should transport goats to Mabvuku, Kadoma,

Damafalls, Epworth and Tafara most of these destinations had minimum transportation cost per goat. A decision tree analysis was developed to determine which choice Joetech company should choose, that is choosing between growing maize or sunflower. Sunflower appeared to be the best choice for the company taking into account the uncertainty of price and weather in Zimbabwe.

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THANK YOU!

Chapter 1

Introduction

Smallholder and family agriculturalists are generally contributing to food protection in the country. There are 1,534,396 smallholder farmers in Zimbabwe. They own 80% of the farm animals and occupy 50% of agricultural land in Zimbabwe. A majority of smallholder farmers have livestock with 65% of them having small livestock such as goats, sheep, and chickens and 45% have each small and large livestock (Mutami, 2015). After the land redistribution programme which gave greater small-scale farmers land, they have been playing a vital role in the growing of crops like cotton and maize and have recently even taken over tobacco production. They are now not solely involved in subsistence farming but they also make contributions to feeding the larger population of Zimbabwe (FOA, 2016). However, although smallholders are key to food security they face numerous challenges such as low revenue, output, changes in market prices and the inability to move with technology.

A discipline called operations research has been used in farm management for the past years. Operations research makes use of mathematical programming to solve many farm-related problems such as the resource allocation problem, what type of crop to grow, water management and making decisions in a risky and uncertain environment. The main study area of concern in farm management is linear programming and dynamic programming (Burt, 1965).

The basis of this research is the application of mathematical ideas to assist smallholder farmers to develop optimal crop production plans, understanding how to make decisions in a risky and uncertain environment and being in a position to compete with large-scale commercial farmers in the country. This chapter, therefore, gives an introduction by revealing the background of the study, statement of the problem, justification of the study, research objectives, research questions, limitations, and delimitation of the research.

1.0.1 Background of study

There are several economic issues because resources are finite in the actual world. As farm resources like land and water grow more and more scarce, economic concerns persist. The farmer also has a finite amount of capital and labor to operate the farm. Similarly, the farmer has limited man-hours and capital to manage the farm. Linear programming can be used to combine the available resources and get the maximum returns from these scarce resources. In other words, linear programming is an advanced approach to farm budgeting since it uses more advanced mathematical concepts to solve farm problems. Linear programming can be used to solve the resource allocation problem, water management and crop management.

Since farmers need to transport their produce and livestock there are interested not only in having an available transportation system but also considering the least amount of time and cost of transportation. Williams et al (2003) postulate that effective transportation systems influence demand by increasing access to the market. Linear programming can be used to find the least cost of transportation. This concept is known as the transportation problem and has been applied in farming for the past years.Other than farming, linear programming has been employed in government, banking, and financial services (Wright, 1996). Additionally, it has enabled companies of all sizes to save billions of dollars (Winston, 1995).

Farmers have no control over some aspects such as prices, weather and unforeseen events but still, they need to make decisions that optimize their returns. Farmers use the knowledge of dynamic programming to make decisions in a risky and uncertain environment.Dynamic programming can be defined as a backward induction technique applied to solve sequential decision problems (Burt, 1965). A decision tree is a useful tool that can be used as a model of sequential decision problems under risk and uncertainty in farming.

Agriculture plays two roles in economic improvement: economic roles and non-economic roles, according to the Food and Agriculture Organization, which stated that agriculture is essential to economic improvement in 2000. Economic functions include generating income, reducing poverty, and ensuring access to food; non-economic roles include managing and conserving natural resources, promoting social cohesion and stability, and preserving cultural legacy. Small-scale farmers will contribute to economic growth if they employ operations research in their farm management.

1.0.2 Problem Statement

Smallholder farmers in Zimbabwe find it difficult to allocate resources and make decisions in a risky and uncertain environment. Their primary objective is to maximize revenue through optimal crop production combinations subject to input constraints. Many farmers engage in farming with traditional knowledge of how to grow crops and estimate input requirements, however, they lack key information on how to allocate resources efficiently to maximize revenue, yield, efficiency, profits and minimize costs. Due to risk and uncertainty in agriculture, farmers have no control over positive variables such as prices, weather, time, and climate change. To uninformed farmers, decision-making is almost impossible. Overall, they find themselves in losses and unable to compete with large-scale farmers.

1.0.3 Research Objectives

The main objective of this study is to develop and apply methods of linear and dynamic programming to solve the important problems faced by small-scale farmers in Zimbabwe. The more specific objectives are:

1. To Identify the causes of failure by smallholder farmers.

2. To solve the land allocation problem in farming.

3. To make decisions in a risk and uncertainty environment.

4. To solve the transportation problem in farming.

5. To clearly show the importance of time in farming.

6. To clearly show the importance of product promotion in farming.

1.0.4 Research Questions

1. What are the main causes of failure by small-scale farmers?

2. What is the important information farmers should have to improve their survival in the agriculture industry?

3. How can small-scale farmers use operations research to solve the land allocation problem in farming?

4. How can small-scale farmers maximize revenue by selling livestock?

5. How can farmers make decisions in a risk and uncertainty environment?

6. How can small-scale farmers use operations research to solve the transportation problem in farming?

7. Revenue maximization under profit constraints in farming?

8. What are the possible production functions for small-scale farmers?

1.0.5 Justification of study/Rationale

The major drive of this research was the continued failure of the smallholder farmers in the industry despite efforts by the government and Food Security Programme (LFSP), banks, and other institutions in gathering resources so that smallholder farmers can invest in farming, access financing, supply productivity-enhancing technologies and engage in non-farm economic activities. Much has been written over the years on problems faced by smallholder farmers which are mainly the lack of finance, lack of better access to markets, low soil fertility, and lack of advanced technology. In Zimbabwe, particularly small-scale farmers, have given less attention to the use of important knowledge such as effective resource allocation and making decisions under risk and uncertainty using mathematical programming knowledge which is important in their success. The research further helps farmers to have a deep understanding of the importance of time in agriculture.

Due to mathematics, statistics and agricultural economics applied in the research, to the researcher, this study partially fulfills the academic requirements to attain the BSc Honours degree in Applied Mathematics with Economics. The economy and the University of Zimbabwe will gain privilege in the area of this research.

1.0.6 Project Layout

The research paper will explore five chapters. The first chapter is an introduction that gives an overview of the concept of the importance of small-scale farmers, land allocation problems, transportation problem and making decisions under risk and uncertainty. This chapter also gives the motivation for the research, goals and the importance of the study. Chapter two discuses literature review of the study. It discusses contributions by previous researchers in the field of study and explains the theoretical concepts involved. Chapter three gives the methodology of the study. This chapter shows how numerous techniques can be used to describe, formulate and solve problems. The softwares to be used are also discussed in this chapter. Data analysis and result interpretation is presented in Chapter four. The obtained results will be presented with the general characteristics of the data and then progressing to the building of the model. Chapter five focuses on discussion of the results,conclusion, limitations, delimitation and recommendations of the research.

Chapter 2

Literature Review

This chapter first discusses the theoretical foundation of farming concepts and then explores the available literature in the area of operations research specifically in farming and related concepts in farming. This section also discusses advanced methods that can be used to solve many complex farm management problems. It suffices to first provide some important terminology to better comprehend the agriculture industry and the problems faced.

2.1 Terminology

Definition 2.1: Agriculture
Agriculture involves crop production, soil cultivation, animal rearing, forestry, and fisheries among other activities, thus agriculture is both an art and an applied science.

Definition 2.2: Operations research
Tulchinsky et al (2014) define operations research as the use of advanced analytical methods to break down complex problems into solvable problems.

Definition 2.3: Production function
This function shows a relationship that converts inputs or resources into outputs. The production function can be written as:

$$Q = f(\mathbf{X})$$

Where
Q is the output or the physical total product (TPP)
$\mathbf{X} = (x_1, x_2, \ldots, x_n)^\top \geq 0$ is a vector of inputs (Debertin, 2012).

In farming, the production function is paramount because it helps the farmer understand how much input is needed to reach a certain level of output or yield. Economists prefer to use continuous production functions instead of discrete production functions because, with a continuous production function, there are no input levels where the yield cannot be calculated.

Definition 2.4: Fixed input

An input that the farmer has no control over. The land owned by a farmer is an example of a fixed input (Debertin, 2012).

Definition 2.5: Variable input

These are inputs that change in relation to the volume of output over a specific time period, such as labor, fertilizers, seeds, and pesticides (Debertin, 2012).

Definition 2.6: Law of diminishing marginal returns(LDMR)

In agriculture economics, the law states that after a certain optimal yield, each extra unit of the variable input added causes a decreasing marginal yield.(Debertin, 2012). As an illustration, consider fertilizing maize with additional kgs of compound D fertilizer. After a certain amount of time, the maize would experience yellowing, wilting, browning of the leaf margins, leaf drop, and slow to no growth, which would result in a lower yield. The law of diminishing returns does not dictate that as more units of a variable input are supplied, the output would decrease overall. The rate of change in the slope or curvature of the production function is described by the Law of Diminishing Marginal Returns. LDMR gives rise to quadratic production function. This law can be illustrated in figure 2.1.

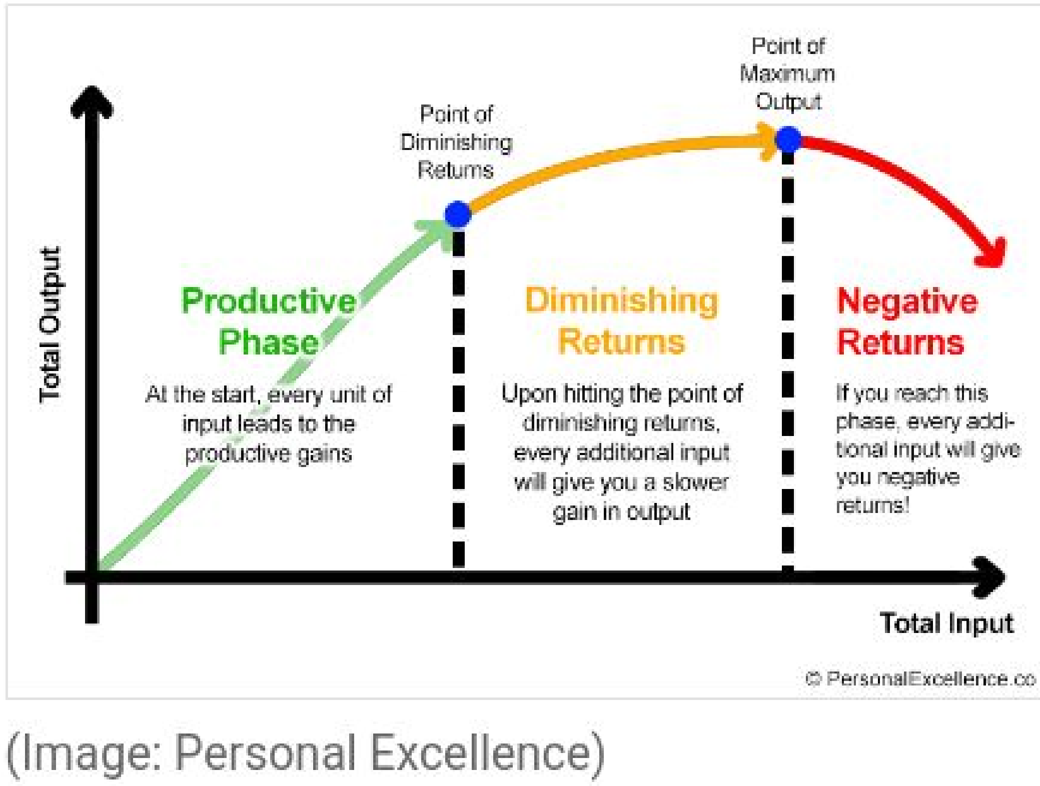


Figure 2.1: Diminishing marginal returns.(Source: Personal Excellence: Online)

Definition 2.7: Assumption of Pure Competition in Farming

This assumption predicts that there are various sellers and buyers in the industry, the business can sell as much as it wants for the going rate and neither a monopoly nor an oligopoly exists.

Definition 2.8: Fixed costs (FC)

The farmer needs to cover these costs whether production takes place or not. Payments for land purchases and depreciation of farm equipment, buildings, and machinery are examples of fixed costs. Fixed costs are equivalent to some fixed monetary amount a since they do not change with output. Thus:

$$FC = a \tag{2.1.1}$$

Definition 2.9: Variable costs (VC) Variable costs change with the quantity of output produced by the farmer. Variable costs include but are not limited to payments for seeds, fertilizers, piece rate labour, fuel, and herbicides. Feed is the main variable cost in the production of livestock. The variable cost function can be expressed as

$$VC = g(Q) \tag{2.1.2}$$

This implies that:

$$\text{Total cost(TC)} = FC + VC = a + g(Q) \tag{2.1.3}$$

Definition 2.10: Cost Function

Assume that each input may be purchased at the going rate in the market r_1 for $x_1; r_2$ for x_2 , Therefore, the cost function is given by:

$$C = r_1x_1 + r_2x_2 \tag{2.1.4}$$

Definition 2.11: Revenue

This is income from selling crops or livestock at the current market price p. The revenue function can be expressed as:

$$R = pQ \tag{2.1.5}$$

The farmer's profit function is given:

$$\Pi = R - TC = R - C = pQ - r_1x_1 - r_2x_2 \tag{2.1.6}$$

Definition 2.12: What does it mean to Optimize

In Latin, the word Optimus means "best" or very good, therefore to optimize is to try to come out with the best or very good solution to the problem at hand. In mathematical optimization, we try to find solutions that maximize or minimize some objective functions, such as minimizing costs of production and maximizing profits.

Definition 2.13: Objective Function

Represents a function that can be maximized or minimized. This function can be expressed as $f(\mathbf{X})$

Definition 2.14: Constraint

An inequality or equation constraint is a restriction on resources or variables of the programming problem

Definition 2.15: Marginal Cost(MC) or Marginal Factor Cost(MFC)

Represents a change in total production cost as a result of producing an extra output (Debertin, 2012). MC can be calculated as:

$$MC = \frac{\partial TC}{\partial Q} \tag{2.1.7}$$

Definition 2.16: Marginal Physical Product(MPP)

This is the change in output or total physical product as a result of a change in a variable input (Debertin, 2012). The marginal physical product can be calculated as:

$$MPP = \frac{\partial Q}{\partial x_i} \quad \text{for} \quad (i = 1, 2, \ldots, n) \tag{2.1.8}$$

Definition 2.17: Marginal Revenue of Output

Represents the change in revenue as a result of a change in output. It is given by:

$$\frac{\partial R}{\partial Q} \tag{2.1.9}$$

Definition 2.18: Value of Marginal Product(VMP)

This is the value of the additional unit of output produced as a result of a further unit of input. (Debertin, 2012). In other words, VMP measures the revenue that can be contributed by the last unit of output. Let p be the current market prices then, VMP is calculated as follows:

$$VMP = pMPP = p\frac{\partial TC}{\partial Q} \tag{2.1.10}$$

Definition 2.19: Constrained Optimization

Constrained optimization problems involve minimization or maximization of an objective function subject to some restrictions. In the real world, resources are limited

and this gives rise to constrained optimization, unlike unconstrained optimization which does not take into account constraints or restrictions. In farming, constrained optimization is of great importance since farmers have limited resources such as land, inputs (fertilizers), capital, time and in this paper we mainly focus on constrained optimization techniques with more than one input.
A constrained optimization problem can be written as:

$$\text{Maximize } f(\mathbf{Y}) \tag{2.1.11}$$

Subject to:

$$\begin{aligned} g(\mathbf{Y}) &\leq \mathbf{b} \\ h(\mathbf{Y}) &= \mathbf{c} \end{aligned} \tag{2.1.12}$$

Where

$$\begin{aligned} \mathbf{Y} &= (y_1, y_2 \ldots, y_n)^\top \\ \mathbf{b} &= (b_1, b_2 \ldots, b_n)^\top \\ \mathbf{c} &= (c_1, c_2 \ldots, c_n)^\top \end{aligned}$$

$f(\mathbf{Y})$ is the objective function, $g(\mathbf{Y})$ is the **inequality constraint** and $h(\mathbf{Y})$ is the **equality constraints**. Constrained optimization problems also involve mixed constraints.

Necessary and Sufficient Conditions for Constrained Optimization

Definition 2.20: Lagrange Multiplier Method
Langrage multipliers give a collection of necessary conditions to select the optimal points of equality-constrained optimization.

Now suppose the farmer wants to maximize revenue from the sale of maize subject to the cost constraint. The problem can be written as:

$$\text{Maximize } f(\mathbf{X}) = pQ \tag{2.1.13}$$

Subject to

$$C = r_1 x_1 + r_2 x_2 \tag{2.1.14}$$

The above problem Equation (2.1.13) and Equation (2.1.14) can be converted to:

$$\begin{aligned} \text{Minimize} \quad \mathcal{L}(\mathbf{X}, \lambda^*) &= f(\mathbf{X}) - \lambda^* (C - r_1 x_1 + r_2 x_2) \\ &= pQ(x_1, x_2) - \lambda^* (C - r_1 x_1 + r_2 x_2) \end{aligned} \tag{2.1.15}$$

$\mathcal{L}(\mathbf{X}, \lambda^*)$ is the Lagrange function and λ^* the Lagrange multiplier(no sign restriction).

Proposition

Suppose $\lambda^* = \lambda_0^*$, maximum $\mathcal{L}(\mathbf{X}, \lambda^*)$ occurs at $X = X_0$ and C is satisfied by X_0, then maximize $\mathcal{L}(\mathbf{X}, \lambda^*)$ = maximize $f(\mathbf{X})$

First-order conditions (FOC) are the necessary conditions for either maximum or minimization of the objective function subject to constraints. The first derivatives of $\mathcal{L}$ with respect to x_1, x_2 and λ^* are equated 0. These conditions are given by:

$$\frac{\partial \mathcal{L}}{\partial x_1} = p\frac{\partial Q}{\partial x_1} - \lambda^* r_1 = ph_1 - \lambda^* r_1 = 0 \tag{2.1.16}$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = p\frac{\partial Q}{\partial x_2} - \lambda^* r_2 = ph_2 - \lambda^* r_2 = 0 \tag{2.1.17}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda^*} = C - r_1 x_1 + r_2 x_2 = 0 \tag{2.1.18}$$

$\frac{\partial Q}{\partial x_1} = h_1 = \text{MPP}\, x_1$ which is the change in output as a input x_1 is varied, holding input x_2 constant

$\frac{\partial Q}{\partial x_2} = h_2 = \text{MPP}\, x_2$ which is the change in output as a input x_2 is varied holding input x_1 constant

Equations (2.1.16) and (2.1.17) can be written as:

$$p\frac{\partial Q}{\partial x_1}/r_1 = p\frac{\partial Q}{\partial x_2}/r_2 = \lambda^* \tag{2.1.19}$$

Now solving the Equations (2.1.18) and (2.1.19) we get the amount of inputs x_1 and x_2 to be employed by the farmer that maximize revenue.

Economic interpretation

Equations (2.1.16) and (2.1.17): state that in order to maximize profits, the value of marginal products must be equal to the cost of inputs.

Equation(2.1.18): indicates that all of the money available to buy x_1 and x_2 will have been spent when the objective function has been maximized. The Lagrange technique demands that each input be purchased with all of the available funds.

Lagrange multiplication (λ^*): The Lagrange multiplier in agriculture can be thought of as the implicit or imputed worth of the last dollar invested in the input.

Definition 2.21: Second Order Conditions(SOC)

FOC of the Lagrangian is necessary but not sufficient conditions. This means that FOC can result in a maximum or minimum. In this example, using SOC we can determine whether the maximized objective function really produce a maximum:

Let $\frac{\partial h_i}{\partial x_j} = h_{ij}$ for $i, j = 1, 2$. Then

$$\frac{\partial}{\partial x_1}(ph_1 - \lambda^* r_1) = ph_{11} \tag{2.1.20}$$

$$\frac{\partial}{\partial x_1}(ph_1 - \lambda^* r_1) = ph_{12} \tag{2.1.21}$$

$$\frac{\partial}{\partial \lambda^*}(ph_1 - \lambda^* r_1) = -r_1 \tag{2.1.22}$$

$$\frac{\partial}{\partial x_1}(ph_2 - \lambda^* r_2) = ph_{21} = ph_{12} \tag{2.1.23}$$

$$\frac{\partial}{\partial x_2}(ph_2 - \lambda^* r_2) = ph_{22} \tag{2.1.24}$$

$$\frac{\partial}{\partial x_1}(ph_1 - \lambda^* r_1) = ph_{12} \tag{2.1.25}$$

$$\frac{\partial}{\partial \lambda^*}(ph_2 - \lambda^* r_2) = -r_2 \tag{2.1.26}$$

$$\frac{\partial}{\partial x_1}(C - r_1 x_1 + r_2 x_2) = -r_1 \tag{2.1.27}$$

$$\frac{\partial}{\partial x_2}(C - r_1 x_1 + r_2 x_2) = -r_2 \tag{2.1.28}$$

$$\frac{\partial}{\partial \lambda^*}(C - r_1 x_1 + r_2 x_2) = 0 \tag{2.1.29}$$

To be guaranteed of a maximum revenue the condition below is required:

$$p\left(2h_{12}r_1r_2 - h_{22}v_1^2 - h_{11}v_2^2\right) > 0 \qquad (2.1.30)$$

Where Equation (2.1.30) is the determinant of the matrix :

$$\mathbf{A} = \begin{pmatrix} ph_{11} & ph_{12} & -r_1 \\ ph_{21} & ph_{22} & -r_2 \\ -r_1 & -r_2 & 0 \end{pmatrix} \qquad (2.1.31)$$

Shortcomings of the Lagrange Method

The Lagrange technique simply assumes that a certain sum of money is available to buy inputs, without providing the farmer any guidance on how much should be spent overall on inputs to optimize revenue (Debertin, 2012).

2.2 Importance of Product Promotion in Farming

Optimization and Product Promotion

Sales are usually affected by promotion, and the farmer may wish to maximize revenue taking into account the expenditure from promoting. Farmers can use the following tools to promote their products or produce

Definition 2.22: Advertising

Advertising, according to Bovee (1992), is the impersonal dissemination of information about goods, services, or concepts through different media. This tool is crucial to a farmer because it enables him or her to display their goods simply and efficiently using text, music, and color to buyers who are located far away (Kotler 2010).

Definition 2.23: Social Media Marketing

This tool enables viral communication among customers across social networks which include Twitter, WhatsApp and Facebook (Pentina et al, 2012). Nowadays farmers use online platforms to market their crops which increases their sales. Social media marketing can also reduce the costs of renting a shop to sell crops because farmers can simply communicate with customers on online platforms and deliver their orders to their doorsteps.

Packaging

Packaging increases the value of the products. Farmers in Zimbabwe can package their produce and can supply directly to large established companies such as Bon Marche and OK.

2.2.1 Maximizing Revenue subject to a Profit Constraint Linear Programming Problems (LPP)

Suppose a farmer produces maize only, let the output be Q and sales are affected by advertising and packaging expenditure a. He wants to maximize revenue R(Q) subject to a profit constraint. The problem becomes:

$$\text{Maximize} \quad R(Q,a) \tag{2.2.1}$$

Subject to

$$\Pi = R(Q,a) - C(Q) - a \geq m \tag{2.2.2}$$

where $C(Q)$ is the cost function, m is the prescribed profit and $Q \geq 0$. Applying the Lagrange and KKT conditions the problem can be solved.

Proposition 3

If marginal revenue and marginal cost are positive ($\frac{\partial R}{\partial Q} > 0, \frac{dC}{dQ} > 0$), the output that maximizes revenue will be such that the profit is equal to the prescribed level a, the marginal revenue $\frac{\partial R}{\partial Q} > 0$ and $\frac{\partial \Pi}{\partial Q} < 0$

Proof

Applying the Lagrange:

$$\mathcal{L} = R(Q,a) + \lambda^*(R(Q,a) - C(Q) - a - m) \tag{2.2.3}$$

Applying KKT conditions:

$$\frac{\partial \mathcal{L}}{\partial Q} = \frac{\partial R}{\partial Q} + \lambda^*(\frac{\partial R}{\partial Q} - \frac{\partial C}{\partial Q}) = (1+\lambda^*)\frac{\partial R}{\partial Q} - \lambda^*\frac{\partial C}{\partial Q} \leq 0 \tag{2.2.4}$$

$$\frac{\partial \mathcal{L}}{\partial a} = \frac{\partial R}{\partial a} + \lambda^*(\frac{\partial R}{\partial a} - 1) = (1+\lambda^*)\frac{\partial R}{\partial a} - \lambda^* \leq 0 \tag{2.2.5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda^*} = R(Q,a) - C(Q) - a - m \leq 0 \tag{2.2.6}$$

Complementary slackness conditions are:

$$Q\frac{\partial \mathcal{L}}{\partial Q} = Q[(1+\lambda^*)\frac{\partial R}{\partial Q} - \lambda^* \frac{\partial C}{\partial Q}] = 0 \tag{2.2.7}$$

$$a\frac{\partial \mathcal{L}}{\partial a} = a[(1+\lambda^*)\frac{\partial R}{\partial a} - \lambda^*] = 0 \tag{2.2.8}$$

$$\lambda^*\frac{\partial \mathcal{L}}{\partial \lambda^*} = \lambda^*[R(Q,a) - C(Q) - a - m] = 0 \tag{2.2.9}$$

$$\lambda^* \geq 0$$

Equation (2.2.8) implies that $\Pi = m$

For $Q \geq 0$ and from equation (2.2.5), then:

$$\frac{\partial R}{\partial a} \leq \frac{\lambda^*}{(1+\lambda^*)} \tag{2.2.10}$$

From our proposition $\frac{\partial R}{\partial a} > 0$, equation (2.2.10) and equation (2.2.8) then $\lambda^* > 0$. We can also write equation (2.2.7) as:

$$\frac{\frac{\partial R}{\partial Q}}{\frac{dC}{dQ}} = \frac{\lambda^*}{1+\lambda^*} \tag{2.2.11}$$

Equation (2.2.11) implies that $\frac{\partial R}{\partial Q} < \frac{dC}{dQ}$ and from our proposition $\frac{dC}{dQ} > 0$ this implies that $\frac{\partial R}{\partial Q} > 0$ and furthermore $\frac{\partial \Pi}{\partial Q} = \frac{\partial R}{\partial Q} - \frac{dC}{dQ} < 0$. Hence the proposition 3 is proved.

2.3 Linear Programming Problems (LPP)

An optimization method called linear programming is used to solve problems where the objective function and the constraints are both linear. George B. Dantzig created the simplex approach and the general LPP in 1947. Linear programming is of paramount importance because it can be applied in many areas such as farming, manufacturing, engineering, statistics, medicine and construction. Other advanced methods under linear programming include the revised simplex method, decomposition method, post-optimal analysis, and Karmarkar's method (Rao, 2009) The LPP can be written as:

$$\text{Maximize } f(\mathbf{X}) = \mathbf{c}^\top \mathbf{X} \tag{2.3.1}$$

Subject to

$$\begin{aligned} \mathbf{AX} &\leq \mathbf{b} \\ \mathbf{X} &\geq \mathbf{0} \end{aligned} \tag{2.3.2}$$

where

$$\mathbf{X} = (x_1, x_2 \ldots, x_n)^\top$$

$$\mathbf{c} = (c_1, c_2 \ldots, c_n) \top$$

$$\mathbf{b} = (b_1, b_2 \ldots, b_n)^\top$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$$

The set of feasible solutions H $= \{\mathbf{X} : \mathbf{AX} = \mathbf{b}, \mathbf{X} \geq \mathbf{0}\}$ is determined by the intersection of the finite set of linear constraints (Luptacik, 2010)

2.3.1 Linear Programming Assumptions

Debertin (2012) identified several LPP assumptions which are:

Linearity or Proportionality

This basic assumption assumes that proportionality exists in the objective and constraints. As an example, if 2 crops sell at $10 then 4 crops would sell for $20. Consequently, if output doubles, profit also doubles..

Additivity

The overall activities are equal to the sum of each individual activity. For instance, the objective function's total revenue is equal to the total revenue contributed by each product individually. Therefore, there is no interaction between the variables used in making decisions..

Continuity

Continuous decision variables exist. This implies that a mix of outputs can be used with both integer and fractional values.

Certainty

Parameters of the objective function coefficients and coefficients of constraint inequalities are known with certainty and might change during the period being studied.

Non-Negative variables

LPP assumes that variables and solutions are always positive, for example, we cannot produce a negative output.

Optimality

In LPP, solutions to the problem always occur at corner points of the set of the feasible solution.

Related Studies for the Land Allocation Problem

Majeke et al (2010) compared the traditional approach of crop planning to the usage of the single-objective linear programming model for land allocation. The traditional method of planning meant that the farmer simply estimate the quantity to grow for different crops or use past experience to make that decision. The researchers made the research with data from a farm in Beatrice in Zimbabwe and used the simplex method to find the optimal crop production plan that would maximize profit. The results obtained clearly show that the LP model gives a better crop production plan than using the traditional method.

Antoine et al (1997) in a paper titled "Multiple criteria land use analysis", the researchers argued that the multi-objective land allocation problem is more realistic than one with a single objective because, in reality, such problems involve a number of competing objectives.

2.3.2 Simplex Method

In standard form, the LPP can be expressed as:

$$\text{Maximize } f(\mathbf{X}) = c_1X_1 + c_2 + \ldots + c_nx_n \tag{2.3.3}$$

subject to

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\leq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\leq b_m \end{aligned} \tag{2.3.4}$$

Where $\mathbf{X} \geq 0$ and $\mathbf{B} \geq 0$. Equation (2.3.5) is created by adding non negative variables $\mathbf{A} = (A_1, A_2 \ldots, A_n)$ called **slack variables** to the left side of each equation because the left side of each inequality of Equation (2.3.4) is less than or equal to the right side. The linear system becomes:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + A_1 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + A_2 &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + A_n &= b_m \end{aligned} \tag{2.3.5}$$

Where $\mathbf{A} \geq 0$

Definition 2.24: Feasible solution
A solution is called feasible if its entries are nonnegative
Definition 2.25: Optimal solution
A feasible solution that optimizes the objective function
Definition 2.26: Optimal basic solution
This is a basic feasible solution for which the objective function is optimal (Rao, 2009)

As an example for illustration, suppose a farmer wants to maximize profit subject to some constraints. The problem is given below:

$$\text{Maximize } 4x_1 + 3x_2 \tag{2.3.6}$$

Subject to

$$5x_1 + x_2 \leq 10 \tag{2.3.7}$$

$$0.5x_1 + x_2 \leq 2 \tag{2.3.8}$$

$$x_1 + 2x_2 \leq 5 \tag{2.3.9}$$

$$\text{Where } \mathbf{X} \geq 0$$

Step 1: Add slack variables to make the system linear. The new problem is:

$$\text{Maximize } A_4 = 4x_1 + 3x_2 \tag{2.3.10}$$

Subject to

$$5x_1 + x_2 + A_1 = 10 \tag{2.3.11}$$

$$0.5x_1 + x_2 + A_2 = 2 \tag{2.3.12}$$

$$x_1 + 2x_2 + A_3 = 5 \tag{2.3.13}$$

$$\text{Where } \mathbf{A} \geq 0$$

Step 2: Express the system as a matrix table or the simplex tabular

The initial basic feasible solution with basic variables is:
$(x_1, x_2, A_1, A_2, A_3, A_4) = (0, 0, 10, 5, 2, 0)$ is but not optimal. The current value of $f(\mathbf{X}) = A_4 = 0$

Step 3: **Optimality Test**: If the bottom row of the tabular has a negative value the solution is not optimal and needs to be improved.

Step 4: **Pivoting to find an improved solution**: To improve the current solution we introduce the entering variable and the leaving variable.

Entering variable (E) is the most negative and smallest entry in the bottom row of the tabular.

Leaving variable (L) is the smallest positive ratio of $\frac{b_i}{a_{ij}}$, in the column determined by the entering variable.

The entry in the tableau in the entering variable's column and the departing variable's row is the pivot element. Applying Gauss Jordan Elimination to the element pivoted. The following iterations are obtained:

Iteration 0

Basic Variables	$x_1(E)$	x_2	A_1	A_2	A_3	A_4	Solution	Min Ratio
$A_1(L)$	5	1	1	0	0	0	10	2
A_2	1	2	0	1	0	0	5	5
A_3	$\frac{1}{2}$	1	0	0	1	0	2	4
A_4	-4	-3	0	0	0	1	0	

Iteration 1

Basic Variables	x_1	$x_2(E)$	A_1	A_2	A_3	A_4	Solution	Min Ratio
x_1	1	$\frac{1}{5}$	$\frac{1}{5}$	0	0	0	2	2
A_2	0	$\frac{9}{5}$	$\frac{-1}{5}$	1	0	0	3	$\frac{5}{3}$
$A_3(L)$	0	$\frac{9}{10}$	$\frac{-1}{10}$	0	1	0	1	$\frac{10}{9}$
A_4	0	$\frac{-11}{5}$	$\frac{4}{5}$	0	0	1	8	

Iteration 2

Basic Variables	x_1	x_2	A_1	A_2	A_3	A_4	Solution
x_1	1	0	$\frac{2}{9}$	0	$\frac{-2}{9}$	0	$\frac{16}{9}$
A_2	0	0	0	1	-2	0	1
x_2	0	1	$\frac{-1}{9}$	0	$\frac{10}{9}$	0	$\frac{10}{9}$
A_4	0	0	$\frac{5}{9}$	0	$\frac{22}{9}$	1	$\frac{94}{9}$

Since the bottom row does not contain any negative value, the solution $(x_1, x_2, A_1, A_2, A_3, A_4) = \left(\frac{16}{9}, \frac{10}{9}, 0, 1, \frac{10}{9}, \frac{94}{9}\right)$ is optimal.

A maximum profit of $\$10, 44$ can be attained.

Remark 1: If the aim is to minimize $f(\mathbf{X})$, then consider instead maximize $-f(\mathbf{X})$

Remark 2: If the constraints of the system include "$\geq$", multiply by -1 to make them "$\leq$" and we apply the method.

2.3.3 Geometry of Linear Programming Problems

Using the graphical technique, feasible solutions and the optimal solution can be visualized. Graphically, the solution to equations (2.3.7) to (2.3.9) is given below:

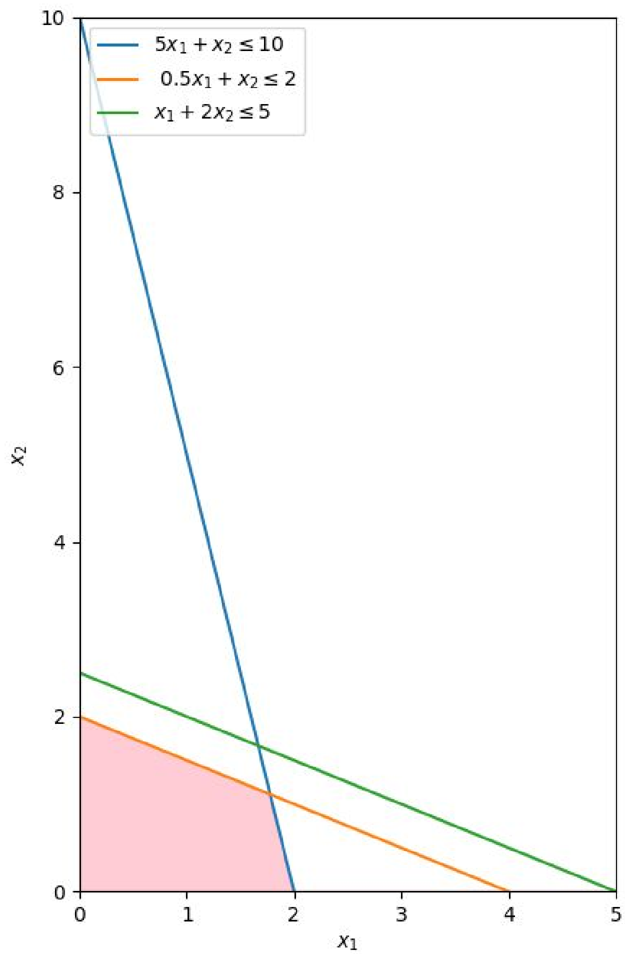


Figure 2.2: Graphical representation

The set of the feasible solution $H = \{(0,0), (0,2), (2,0), (\frac{16}{9}, \frac{10}{9})\}$

2.3.4 Duality

Primal Problem

$$\text{Maximize } f(\mathbf{X}) = \mathbf{c}^\top \mathbf{X} \tag{2.3.14}$$

Subject to

$$\begin{aligned} A\mathbf{X} &\leq \mathbf{b} \\ \mathbf{X} &\geq 0 \end{aligned} \tag{2.3.15}$$

Dual Problem

$$\text{Minimize } f(\mathbf{Y}) = \mathbf{Y}^\top \mathbf{b} \tag{2.3.16}$$

Subject to

$$\mathbf{Y}^\top A \geq \mathbf{c}^\top \tag{2.3.17}$$

$$\mathbf{Y}^\top \geq 0$$

Where $\mathbf{Y} = (y_1, y_2, \ldots, y_n)^\top$

2.3.5 Implications of Linear Programming Assumptions in Farming

The assumption of linearity or proportionality is unrealistic. This assumption implies constant returns to scale ie all the inputs must be used in the given fixed proportion. But in reality, relations are not always linear, that is in farming we can have decreasing and increasing returns. If the objective function (2.3.14) is a linear profit function then $\frac{\partial f}{\partial x_j} = c_j$ for $j = 1, 2 \ldots, n$ implies that the profit generated by every (additional) unit of crop j is the same. This assumption implies a perfect competition market (Luptacik, 2010). However, imperfect competition markets exist. In conclusion, linear programming should be used when the assumptions hold otherwise non-linear programming can be used.

2.3.6 Two Phase Method

The method makes make use of two phases to solve the LLP when a initial feasible solution is not readily present. Phase 1 can be used to find the basic feasible solution and if we get one, Phase 2 is applied to solve the original problem.

- **Phase I**

  Step 1: To find the initial basic solution, the problem in converted into an equation form and artificial variables added to the corresponding constraints.
  Step 2: We find a initial solution of the corresponding equations that, regardless of whether or not it is a minimization or maximization LPP that we are guaranteed it minimizes the sum of the artificial variables.
  Step 3: If an artificial variable has a positive value in the optimal solution, the original problem is infeasible, stop the process. Otherwise, go to Phase II.

- **Phase II**: Solve the original problem, beginning with the initial feasible solution obtained in phase I (Hamdy, 2007)

2.4 The Transportation Problem

2.4.1 Past Researches in Transportation Problem

Sharmal et al (2012) collected data from Albert David company in India to reduce the transportation cost of moving trucks from the plant (source) to warehouses (destination). Trucks are transported from Madideep (A), Gajiabad (B) and Calcutta to Bhopal (X), Raipur (Y) and Mumbai. The problem was solved by the following methods, Vogel approximation, Big -M , Two-phase and Dual simplex. The methods give the same optimal solution although the allocations of trucks to different warehouses are different.

Brown et al (1987) advanced the transportation problem and developed a mixer integer model for multi-commodity systems. The model aims to minimize production, transporting and fixed assignment costs.

The transportation problem was advanced further by Selim et al (2006). In this article, a supply chain distribution network is created to choose the optimal quantities, locations, and warehouse capacity levels to deliver goods to retailers at the lowest possible cost while also providing the appropriate level of service to retailers.

2.4.2 Transport Problem Formulation

In order to minimize transportation costs for industries with sources and destinations while satisfying demand and supply restrictions, a particular class of linear programs known as the transportation problem is used. F.L Hitchcock was the first to present the transportation problem in 1941 and after his work, other scholars contributed and reduced the problem to the well know transportation problem model,

Given m sources and n destinations, the transportation problem is given as a LP model:

$$\text{Minimize } z(\mathbf{X}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{2.4.1}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq s_i, \quad (i = 1, 2, \ldots, n) \tag{2.4.2}$$

$$\sum_{i=1}^{m} x_{ij} \geq d_i, \quad (i = 1, 2, \ldots, m) \tag{2.4.3}$$

$$x_{ij} \geq 0, \quad \forall i, j$$

Assumptions

1. The problem should be balanced.
2. No more quantity of goods leave the source than there are in stock.
3. The demand of each destination is satisfied

Balanced Transportation Problem

The demand and supply are equal.

Unbalanced Transportation Problem

Demand and supply are not equal. We add a dummy column or row to make it a balanced transportation problem.

The basic steps in solving the transport problem are;

Step 1. Finding the initial feasible solution.
Step 2. Finding the optimal solution using the initial solution.

In step 1, the transportation problem uses three methods that can quickly find the initial feasible solution. The commonly used heuristic methods are:

1. Northwest-corner method
2. Least-cost method
3. Vogel approximation method

The initial feasible solution produced by the three approaches differs in terms of "quality" where a better initial feasible solution results in a lower objective value. The Vogel Approximation approach typically, but not always, produces the best initial basic solution.

After step 1, the UV/MODI Method is used to find the **optimal solution** to the transportation problem.

2.4.3 Northwest-Corner Method

The basic variables are selected from the extreme left corner.

Step 1: Select the northwest corner cell and assign the minimum value of the corresponding cell's supply and demand.
Step 2: Subtract the above minimum value from the corresponding cell's supply and demand. At this step, we get three scenarios.

- If the supply is equal to 0, we strike that row and move to the next cell. If demand equals 0, we strike that column and move right to the next cell.

- If supply and demand are 0, then we strike both the row and the column and move diagonally to the next cell.

Step 3: We repeat the above steps until all the supply and demand values are 0.

2.4.4 Least Cost Method

In this method, the allocation starts with the cell with the lowest value (ties are broken arbitrarily). After the allocation, the satisfied row (column) is removed and the amounts of supply and demand quantities are modified accordingly. Only one is crossed out if both the row and the column are satisfied at the same time. Repeat the process until only one row (column) is left uncrossed out by looking for the uncrossed-out cell with the lowest unit cost (Mishra, 2017).

2.4.5 Vogel Approximation Method(VAM)

The initial feasible solutions produced by this method are better than those of the least-cost method, though this is not always the case. Because the initial basic solution is either an optimal solution or very close to the optimal solution, it is much preferred over the other methods above. According to Korukoglu and Balli (2011) the VAM steps are:

Step 1: Balance the transportation problem if supply and demand are not equal
Step 2 Determine the penalty cost for each row(column) by subtracting the lowest cell cost in the row (column) from the next lowest cell cost in the same row (column).
Step 3: Select the row (column) with the highest penalty cost
Step 4: Allocate as much as possible to the feasible cell with the lowest transportation cost in the row (column) with the highest penalty cost.
Step 5: Repeat steps 2,3 and 4 until all requirements have been met.
Step 6: Compute the total transportation cost for the feasible allocations.

2.5 Importance of Time in Farming

Most successful farmers are time cautious, they understand when and what to grow during different seasons of the year. Different crops perform well in different seasons, for instance, maize needs to be cultivated in regions where the mean daily temperature is less than 19°C or where the mean of the summer months is less than 23°C. Between 350 and 450mm of rain is needed to provide a yield of 3152kgs per ha (Jean du Plessis, 2003). In Zimbabwe, maize is mostly grown between October and December.

2.5.1 Time as an Input in Farming

In farming, time can be seen or considered an input. In order to optimize revenues and minimize costs, the farmer needs to carefully allot time for land preparation, planting, and harvesting. Weather conditions limit the amount of time needed for land preparation, planting, and harvesting, and field time used for one activity cannot be used again for another activity.

As an illustration, consider a farmer who has finite amount of hours **T** available during a single production season suitable for the land preparation (Lp), planting (Tp) and harvest (Th). Assume that the farmer grows three crops, maize, potatoes and wheat. The farmer has already decided the hectares for Q_1, Q_2 and Q_3 to be grown and wishes to allocate available time to each crop (Debertin, 2012)

Revenue

$$\mathrm{R} = p_1 Q_1 + p_2 Q_2 + p_3 Q_3$$

where

P is the respective market price for each crop.

Q is the amount of output for each crop.

t_{p1}, t_{p2}, t_{p3} are times for planting maize, potatoes and wheat respectively.

$t_{lp1}, t_{lp2}, t_{lp3}$ are times for land preparation for maize, potatoes and wheat respectively.

t_{h1}, t_{h2}, t_{h3} are times for harvesting maize, potatoes and wheat respectively.

Output for crop i is a relationship of the times available in planting, land preparation and harvest, that is:

$$Q_1 = Q_1 \left(tp_1, lp_1, h_1 \right)$$

$$Q_2 = Q_2\left(tp_2, lp_2, h_2\right)$$

$$Q_3 = Q_3\left(tp_3, lp_3, h_3\right)$$

The objective of farmer is to maximize revenue subject to time restrictions or constraints. Thus, the LP is given by:

$$\begin{aligned} \text{Maximize} \quad R = & p_1 Q_1\left(t_{p1}, t_{lp1}, t_{h1}\right) \\ & + p_2 Q_2\left(t_{p2}, t_{lp2}, t_{h2}\right) + p_3 Q_3\left(t_{p3}, t_{lp3}, t_{h3}\right) \end{aligned} \tag{2.5.1}$$

Subject to

$$t_{p1} + t_{p2} + t_{p3} \leq T_P \tag{2.5.2}$$

$$t_{lp1} + t_{lp2} + t_{lp3} \leq T_{LP} \tag{2.5.3}$$

$$t_{h1} + t_{h2} + t_{h3} \leq T_H \tag{2.5.4}$$

Introducing the lagrangian multipliers λ^*, μ^* and Φ which are values, then the lagrangian function is given by:

$$\begin{aligned} \mathcal{L} = & p_1 Q_1\left(t_{p1}, t_{lp1}, t_{h1}\right) + p_2 Q_2\left(t_{p2}, t_{lp2}, t_{h2}\right) + p_3 Q_3\left(t_{p3}, t_{lp3}, t_{h3}\right) \\ & + \lambda^*\left(T_P - t_{p1} - t_{p2} - t_{p3}\right) + \mu^*\left(T_{LP} - t_{lp1} - t_{lp2 - t_{lp3}}\right) \\ & + \Phi^*\left(T_H - t_{h1} - t_{h2} - t_{h3}\right) \end{aligned} \tag{2.5.5}$$

The first-order conditions(FOC) are:

$$p_1\frac{\partial Q_1}{\partial t_{p1}} = \lambda^* \tag{2.5.6}$$

$$p_2\frac{\partial Q_2}{\partial t_{p2}} = \lambda^* \tag{2.5.7}$$

$$p_3\frac{\partial Q_3}{\partial t_{p3}} = \lambda^* \tag{2.5.8}$$

$$p_1\frac{\partial Q_1}{\partial t_{lp1}} = \mu^* \tag{2.5.9}$$

$$p_2\frac{\partial Q_2}{\partial t_{lp2}} = \mu^* \tag{2.5.10}$$

$$p_3\frac{\partial Q_3}{\partial t_{lp3}} = \mu^* \tag{2.5.11}$$

$$p_1\frac{\partial Q_1}{\partial t_{h1}} = \Phi^* \tag{2.5.12}$$

$$p_2\frac{\partial Q_2}{\partial t_{h2}} = \Phi^* \tag{2.5.13}$$

$$p_3\frac{\partial Q_3}{\partial t_{h3}} = \Phi^* \tag{2.5.14}$$

$$T_P - t_{p1} - t_{p2} - t_{p3} = 0 \tag{2.5.15}$$

$$T_{LP} - t_{lp1} - t_{lp2 - t_{lp3}} = 0 \tag{2.5.16}$$

$$T_{h1} - t_{h2} - t_{h3} = 0 \tag{2.5.17}$$

From (2.5.6) to (2.5.14), it implies:

$$p_1\frac{\partial Q_1}{\partial t_{p1}} = p_2\frac{\partial Q_2}{\partial t_{p2}} = p_3\frac{\partial Q_3}{\partial t_{p3}} = \lambda^* \tag{2.5.18}$$

$$p_1\frac{\partial Q_1}{\partial t_{lp1}} = p_2\frac{\partial Q_2}{\partial t_{lp2}} = p_3\frac{\partial Q_3}{\partial t_{lp3}} = \mu^* \tag{2.5.19}$$

$$p_1\frac{\partial Q_1}{\partial t_{h1}} = p_2\frac{\partial Q_2}{\partial t_{h2}} = p_3\frac{\partial Q_3}{\partial t_{h3}} = \Phi^* \tag{2.5.20}$$

Solving equations (2.5.15),(2.5.16),(2.5.17),(2.5.18),(2.5.19) and (2.5.20) we get the time that needs to be allocated to different crops and activities.

2.5.2 Economic interpretation of langragean multipliers

The assumed values for an additional unit of available time in the planting, land preparation, or harvest seasons for each crop, in terms of revenue to the farm. The production of different crops is severely constrained by the available time when the langragean multipliers have big values for a given period. Even though time in that particular period has some positive value in the production process, a Lagrangian multiplier that is close to zero suggests that it does not pose a significant constraint (Debertin, 2012).

2.6 Non-Linear Programming

In 1948 famous mathematicians and economists attended a meeting of the Econometric Society in Wisconsin where linear programming was presented by Dantzig, they also constructively agreed that real-world problems are not always linear. This gave rise to a non-linear programming paper by two mathematicians Kuhn and Tucker. In actuality, linear problems have been viewed as specialized non-linear problems. Convex programming, quadratic programming, separable programming, geometric programming, and fractional programming are examples of non-linear programming problems (Luptacik, 2010).

2.6.1 Non-Linear Problems in Agriculture

In agriculture, we are also faced with many non-linear problems for example we might need to minimize a quadratic or cubic cost function subject to some constraints. In general, most functions are quadratic. Small-scale farmers need to have an understanding of non-linear programming.

2.6.2 Quadratic Programming Problem (QPP)

Definition 2.27

A function is called a quadratic form if it can be expressed in the form

$$f(X) = X^\top DX \tag{2.6.1}$$

Where

$$X = (x_1, x_2, \ldots, x_n)^\top$$
$$D_i = \begin{pmatrix} d_{11} & d_{12} & \ldots & d_{1n} \\ d_{21} & d_{22} & \ldots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \ldots & d_{mn} \end{pmatrix}$$

1. If $f(X) > 0, \forall X \neq 0$ then the quadratic form is positive definite and thus strictly convex.

2. If $f(X) \geq 0, \forall X \neq 0 : \exists$ one $X \neq 0$ satisfying $f(X) = 0$, then the quadratic form is positive semi-definite. The quadratic form is a convex function.

3. If $f(X) < 0$, the quadratic form is negative definite and a concave function.

4. If $f(X) \leq 0$, the quadratic form is negative semi-definite.

5. The quadratic form is indefinite if it is none of the above types.

Proposition

The quadratic form $X^\top DX$ is positive definite if and only if $D_i > 0$ for $i = 1, 2 \ldots n$ where

$$D_i = \begin{pmatrix} d_{11} & d_{12} & \ldots & d_{1n} \\ d_{21} & d_{22} & \ldots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \ldots & d_{mn} \end{pmatrix} \tag{2.6.2}$$

2.6.3 Quadratic Programming Problem Formulation

The QPP can be formulated as:

$$\text{Minimize}\, z(\mathbf{X}) = \frac{1}{2}\mathbf{X}^\top\mathbf{D}\mathbf{X} + \mathbf{P}^\top\mathbf{X} \tag{2.6.3}$$

Subject to

$$\begin{aligned} \mathbf{AX} &\leq \mathbf{b} \\ \mathbf{A} &\geq \mathbf{0} \end{aligned} \tag{2.6.4}$$

where

$$\begin{aligned}
\mathbf{X} &= (x_1, x_2 \ldots, x_n)^\top \\
\mathbf{P} &= (p_1, p_2 \ldots, p_n) \\
\mathbf{b} &= (b_1, b_2 \ldots, b_n)^\top \\
\mathbf{A} &= \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix} \\
\mathbf{D} &= \begin{pmatrix} d_{11} & d_{12} & \ldots & d_{1n} \\ d_{21} & d_{22} & \ldots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \ldots & d_{mn} \end{pmatrix}
\end{aligned}$$

PX is the linear part, $\mathbf{X}^\top\mathbf{DX}$ is the quadratic part and constraints are of linear type. $\mathbf{D} = \mathbf{D}^\top$ (symmetric) and positive definite. To solve the problem (2.6.3) and (2.6.4), we apply the KKT conditions.

2.6.4 Kuhn Tucker Conditions

By introducing m additional slack variables $s_i^2, i = 1, 2 \ldots, m$ and surplus variables $r_j^2, j = 1, 2 \ldots, n$, the mathematical problem modifies to:

$$\text{Minimize}\, z(\mathbf{X}) = \frac{1}{2}\mathbf{X}^\top\mathbf{DX} + \mathbf{P}^\top\mathbf{X} \tag{2.6.5}$$

Subject to

$$\begin{aligned} \mathbf{A}_i^\top\mathbf{X} + s_i^2 &= b_i, \quad (i = 1, 2 \ldots, m) \\ -x_j + r_j^2 &= 0, \quad (j = 1, 2 \ldots, n) \end{aligned} \tag{2.6.6}$$

Applying the Langrage method, the Langrage function is:

$$\mathcal{L}(\mathbf{X}, \mathbf{S}, \mathbf{R}, \lambda, \mu) = \frac{1}{2}\mathbf{X}^\top\mathbf{DX} + \mathbf{P}^\top\mathbf{X} + \sum_{i=1}^{m} \lambda_i^* \left(\mathbf{A}_i^\top\mathbf{X} + s_i^2 - b_i\right) + \sum_{j=1}^{n} r_i \left(-x_j + r_j\right) \tag{2.6.7}$$

The KKT conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial x_j} = c_j + \sum_{i=1}^{n} d_{ij} x_i + \sum_{j=1}^{m} \lambda^* a_{ij} - \mu^* = 0, \quad (j = 1, 2 \ldots, n) \tag{2.6.8}$$

$$\frac{\partial \mathcal{L}}{\partial s_j} = 2\lambda_i^* s_i = 0, \quad (i = 1, 2 \ldots, m) \tag{2.6.9}$$

$$\frac{\partial \mathcal{L}}{\partial r_j} = 2\mu_i^* r_j = 0, \quad (j = 1, 2 \ldots, n) \tag{2.6.10}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i^*} = \mathbf{A}_i^\top \mathbf{X} + s_i^2 - b_i = 0, \quad (i = 1, 2 \ldots, m) \tag{2.6.11}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_i^*} = -x_j + r_j^2 = 0, \quad (j = 1, 2 \ldots, n) \tag{2.6.12}$$

Let $s_i^* = s_i^2 \geq 0, i = 1, 2 \ldots, n$ be the new variable, then equation(2.6.11) can be written as

$$\mathbf{A}_i^\top \mathbf{X} - b_i = -s_i^2 = s_i^* \tag{2.6.13}$$

Multiplying Equation (2.6.9) by s_i and Equation (2.6.10) by r_j we get:

$$\begin{aligned} \lambda_i^* s_i^2 &= \lambda_i^* s^* \\ &= \lambda_i^* \left(\mathbf{A}_i^\top \mathbf{X} - b_i \right) = 0, \quad (i = 1, 2, \ldots, m) \end{aligned} \tag{2.6.14}$$

$$\mu_j^* r_j^2 = 0, \quad (j = 1, 2 \ldots, n) \tag{2.6.15}$$

From Equation (2.6.12)

$$\mu_j^* x_j = 0, \quad (j = 1, 2 \ldots, n) \tag{2.6.16}$$

$$x_i \geq 0, \quad (i = 1, 2 \ldots, n) \tag{2.6.17}$$

$$s_i^* \geq 0, \quad (i = 1, 2 \ldots, n) \tag{2.6.18}$$

$$\lambda_i^* \geq 0, \quad (i = 1, 2 \ldots, n) \tag{2.6.19}$$

$$\mu_j^* \geq 0, \quad (j = 1, 2 \ldots, m) \tag{2.6.20}$$

$$\lambda_i^* s_i^* \geq 0, \quad (i = 1, 2 \ldots, n) \tag{2.6.21}$$

$$\mu_j^* x_j \geq 0, \quad (j = 1, 2 \ldots, m) \tag{2.6.22}$$

If we introduce artificial variables $A_i > 0$ to Equation (2.6.8), the problem is converted into an LPP, that is the new problem is:

$$\text{Minimize } w = \sum_{j=1}^{n} A_i \tag{2.6.23}$$

Subject to

$$c_j + \sum_{i=1}^{m} d_{ij} + \sum_{j=1}^{n} \lambda_i^* d_{ij} - \mu_i^* + A_j = 0, \quad (j = 1, 2 \ldots, n) \tag{2.6.24}$$

$$\mathbf{A}_i^\top \mathbf{X} + s_i^* = b_i \tag{2.6.25}$$

$$\mathbf{X} \geq 0$$

$$\lambda^* \geq 0$$

$$s^* \geq 0$$

$$\mu^* \geq 0$$

Equations (2.6.21 and 2.6.22), which are complimentary slackness equations, must be satisfied by the solution. To obtain an optimal solution to the LPP, which then becomes the optimal solution to the QPP, we use the two-phase simplex method previously stated (Singiresu, 2009).

2.6.5 Quadratic Programming in Farming Example

Suppose that the farmer employs two resources or inputs x_1 and x_2 to make one output. He wishes to minimize a quadratic cost function subject to a linear production function and another linear constraint. Let the problem be expressed as;

$$\text{Minimize } C(\mathbf{X}) = 6x_2^2 + 2x_1^2 - 2x_1x_2 - 2x_1 - 2x_2 \tag{2.6.26}$$

subject to

$$2x_1 + x_2 \leq 6 \tag{2.6.27}$$

$$x_1 - 4x_2 \leq 0 \tag{2.6.28}$$

$$x_1, x_2 \geq 0$$

The problem is a QPP

In matrix form Eqn (2.6.5) to (2.6.28) can be written as;

$$\text{Minimize} \quad C(X) = (-2,-2)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \frac{1}{2}(x_1, x_2)\begin{pmatrix} 4 & -2 \\ -2 & 12 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.6.29}$$

subject to

$$\begin{pmatrix} 2 & 2 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} S_1^* \\ S_2^* \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \tag{2.6.30}$$

$$-x_1 + \mu_1^* = 0 \tag{2.6.31}$$

$$-x_2 + \mu_2^* = 0 \tag{2.6.32}$$

where $S_i^* = S_i^2$ and $\mu_i^* = \mu_i^2$ for $i = 1, 2$

$$C^\top = \begin{pmatrix} -2 \\ -2 \end{pmatrix}, \quad D = \begin{pmatrix} 4 & -2 \\ -2 & 12 \end{pmatrix}, \quad A = \begin{pmatrix} 2 & 1 \\ 1 & -4 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

Applying KKT conditions of the equations we obtain the following problem:

$$\text{Minimize} \quad w = A_1 + A_2 \tag{2.6.33}$$

subject to

$$
\begin{aligned}
4x_1 - 2x_2 + 2\lambda_1^* + \lambda_2^* - \mu_1^* + A_1 &= 2 \qquad (2.6.34)\\
-2x_1 + 12x_2 + \lambda_1^* - 4\lambda_2^* - \mu_2^* + A_2 &= 2 \qquad (2.6.35)\\
2x_1 + x_2 + S_1^* &= 6 \qquad (2.6.36)\\
x_1 - 4x_2 + S_2^* &= 0 \qquad (2.6.37)\\
\lambda_i^* S_i^* &= 0 \qquad (2.6.38)\\
\mu_i^* x_i &= 0 \qquad (2.6.39)\\
x_i &\geq 0\\
S_i^* &\geq 0\\
\lambda_i^* &\geq 0\\
\mu_i^* &\geq 0, \quad (i = 1, 2)
\end{aligned}
$$

The problem is now converted to a LPP and using Phase 1, the following iteration are obtained:

Iteration 0

Destination

Basic Variables	x_1	x_2	λ_1^*	λ_2^*	μ_1^*	μ_2^*	A_1	A_2	S_1^*	S_2^*	Solution	Min Ratio
A_1	4	-2	2	1	-1	0	1	0	0	0	2	
A_2	-2	12	1	-4	0	-1	0	1	0	0	2	$\frac{1}{6}$
S_1^*	2	1	0	0	0	0	0	0	1	0	6	6
S_2^*	1	-4	0	0	0	0	0	0	0	1	0	
$-w$	-2	-10	-3	3	1	1	0	0	0	0	-4	

Iteration 1

Destination

Basic Variables	x_1	x_2	λ_1^*	λ_2^*	μ_1^*	μ_2^*	A_1	A_2	S_1^*	S_2^*	Solution	Min Ratio
A_1	$\frac{11}{3}$	0	$\frac{13}{6}$	$\frac{1}{3}$	-1	$\frac{-1}{6}$	0	$\frac{1}{6}$	0	0	$\frac{7}{3}$	$\frac{7}{11}$
A_2	$\frac{-1}{6}$	1	$\frac{1}{12}$	$\frac{-1}{3}$	0	$\frac{-1}{12}$	0	$\frac{1}{12}$	0	0	$\frac{1}{6}$	
S_1^*	$\frac{13}{6}$	0	$\frac{-1}{12}$	$\frac{1}{3}$	0	$\frac{1}{12}$	0	$\frac{-1}{12}$	1	0	$\frac{35}{6}$	$\frac{35}{13}$
S_2^*	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{-4}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	1	$\frac{2}{3}$	2
$-w$	$\frac{-11}{3}$	0	$\frac{-13}{6}$	$\frac{-1}{3}$	1	$\frac{1}{6}$	0	$\frac{-1}{6}$	0	0	$\frac{-7}{11}$	

Iteration 2

Destination

Basic Variables	x_1	x_2	λ_1^*	λ_2^*	μ_1^*	μ_2^*	A_1	A_2	S_1^*	S_2^*	Solution
x_1	1	0	$\frac{13}{22}$	$\frac{1}{11}$	$\frac{-3}{11}$	$\frac{-1}{11}$	0	$\frac{1}{11}$	0	0	$\frac{7}{11}$
x_2	0	1	$\frac{2}{11}$	$\frac{-7}{22}$	$\frac{-1}{22}$	$\frac{-13}{132}$	0	$\frac{13}{132}$	0	0	$\frac{3}{11}$
S_1^*	0	0	$\frac{-15}{11}$	$\frac{3}{22}$	$\frac{13}{22}$	$\frac{37}{132}$	0	$\frac{-37}{132}$	1	0	$\frac{49}{11}$
S_2^*	0	0	$\frac{3}{22}$	$\frac{-15}{11}$	$\frac{1}{11}$	$\frac{4}{11}$	0	$\frac{10}{33}$	0	1	$\frac{5}{11}$
$-w$	0	0	0	0	0	0	1	1	0	0	0

Solutions are $x_1^* = \frac{7}{11}, \quad x_2^* = \frac{3}{11}$ (Basic Variables)

The smallest possible total cost for producing the output is $C^*(X^*) = \frac{-10}{11}$

2.6.6 Farmers Making Decisions under Risk and Uncertain Environment

Farmer's success is greatly affected by the environment in which they operate. Farming is risky, one cannot be certain how things turn out, however, farmers should make rational decisions daily.

Definition 2.28: Probability

Probability refers to the likelihood that an outcome will occur.

Definition 2.29: Risky Environment

In a risk environment, outcomes and probability of occurrence are both either known or may be approximated, according to Knight (1921).

2.6.7 Sources of Risk in Farming

Kahan (2008) noted that sources of risk are mainly from five sources:

Definition 2.30: Production Risk

Crop output is highly influenced by weather , pests and diseases. Low or high rainfall and drought affect crop production. Livestock production also affected by diseases. Contagious infections may have an impact on labour, decreasing output.

Definition 2.31: Marketing Risk

Farmers has no control over prices changes in the market. Production cost, and product prices fluctuate making it difficult to make decisions.

Definition 2.32: Financial Risk

Farmers borrow funds to finance operations. There is a risk due to the unpredictability of future interest rate levels, the lender's willingness and capacity to extend credit, and the farmer's capacity for payback.

Definition 2.33: Institutional Risk

There is no guarantee that institutions that support farming will continue providing funding or services to farmers. Government policies such as subsidies affect farming.

Definition 2.34: Human Risk

The risks to the farm business posed by illness or death are referred to as human risks. Farm activities can be disrupted by illness and death.

Attitude of the Farmer Toward Risk

Farmers may be divided into three types: risk-neutral; risk-takers and risk-averse.

Definition 2.35: Risk-Averse

Farmers prefer outcomes with low uncertainty to those outcomes with high uncertainty. Risk-averse farmers usually insure against losses.

Definition 2.36: Risk Taker

These farmers are open to riskier economic opportunities and are not afraid to take calculated risks. **Definition 2.37: Risk Neutral**

These farmers are indifferent between taking and avoiding risks.

2.6.8 Other Strategies to deal with Risk and Uncertainty

Other approaches are available for farmers to use to reduce the effects of risk and uncertainty. When appropriately applied, each of these tactics decreases losses. According to Debertin (2012), other strategies that a farmer may employ are:

Have Insurance

Insurance policies ensure that the cost of coverage (insurance premium) is reasonable in comparison to any potential loss. Small-scale farmers should invest in crop insurance plans, asset insurance and labour insurance.

Contractual Arrangements

Farmers can enter into contracts on the futures market to sell a certain commodity at a specific price for delivery at a future date. A price to be paid after the harvest should be specified in the contract at the start of the season to prevent price uncertainty.

Diversification

Farmers are urged to vary their crop portfolios so that crop production losses in one area can be more than offset by crop production gains in another area.

Definition 2.38: Uncertain Environment

According to Frank Knight (1921), outcomes and their corresponding probabilities of occurring were unknown in an uncertain environment.

2.7 Dynamic Programming

According to Burt (1965),dynamic programming can be defined as a backward induction technique applied to complex sequential decision problems and reduces the problems into simpler versions. Methods that can be adopted in making decisions under a risk and uncertain environment are decision tree diagrams and bayes rule.

2.7.1 Bayes Rule /Expected Income Method

The farmers can assign probabilities to each outcome. To calculate the expected income, payoffs are weighted with the respective probabilities (Debertin, 2012). Suppose a farmer can either grow maize or wheat this year, given the information below we can find the expected income for the two strategies:

Table 2.1: Expected Payoff

Action	High Yield	Low Yields
Probability	0.5	0.5
Grow maize($)	10000	5000
Grow wheat($)	15000	4000

If the farmer grows maize, the expected income is $0.5 \times \$10000 + 0.5 \times \$5000 = \$7500$
If the farmer grows wheat, the expected income is $0.5 \times \$15000 + 0.5 \times \$4000 = \$9500$
If the farmer's objective is to maximize expected income the best decision is to grow wheat and expect a return of $9500.

2.7.2 Decision Trees

Decision trees visualize the problem using probability trees and the expected payoffs to determine the optimal conclusion. The decision trees make use of the following:

Decision folk - this indicates a decision has to be made and is represented by a small rectangle.
Chance folk - it indicates a random event occurring and it is represented by a small circle.
Probabilities-the farmer places probabilities on random events that may occur.
Conditional Probabilities
Let A, and B be two events. The conditional probability of A given B, denoted by

$P(A \mid B)$ can be defined as:

$$P(A \mid B) = \frac{(A \cap B)}{P(B)} \tag{2.7.1}$$

$$\text{Similarly,} \quad P(B \mid A) = \frac{(B \cap A)}{P(A)} \tag{2.7.2}$$

$$\begin{aligned} \text{Thus,} \quad P(A \cap B) &= P(B \cap A) \\ &= P(A \mid B)P(B) \\ &= P(B \mid A)P(A) \end{aligned} \tag{2.7.3}$$

Backward Induction

This is a backward iterative process and also makes use of the expected monetary value method. An application of decision trees and a backward induction process will be discussed in Chapter 4.

2.7.3 Past Researches for Decision Trees Analysis

Veenadhara et al (2011) in the journal paper titled “Soybean Productivity Modeling using Decision Tree Algorithm”, used Bhopal district (India) meteorological data to model the impact of climatic variables on soybean production using the decision tree algorithm. A sample of climatic information for the period 1984 to 2003 was collected. The study shows the effects of rainfall, evaporation, maximum relative humidity and maximum temperature on soybean yield using regression analysis. In each year, each input variable is classified as high or low based on average values. The data is also classified using decision tree analysis and the gain information is obtained for each variable.The decision tree analysis revealed that maximum relative humidity, followed by temperature and rainfall, had the greatest impact on soybean. The algorithm formed from the decision tree help in the prediction of the factors influencing the productivity of the soybean crop depending on the climate variables.

Rackha et al (2021) used machine learning for large data to develop a decision tree analysis and predict the weather conditions, soil conditions and prices of crops by comparing them with the past year’s prices.

Advantages of Tree Diagrams

1. Potential options and choices are considered at the same time.

2. Use of probabilities enables the “risk” of the options to be addressed

3. Likely costs are considered as well as potential benefits.

Disadvantages of Tree Diagrams

1. Probabilities are just estimates always prone to error.

2. Make use of quantitative data and ignore the qualitative aspect. The techniques do not reduce the amount of risk.

Chapter 3

Methodology

This chapter introduces the various approaches that will be used in the building of four models:

1. Linear programming to solve the Land allocation and optimization problem.
2. Linear programming to maximize the revenue from sale of livestock.
3. Transportation problem in agriculture
4. Decision making using decision tree analysis.

The chapter tries to show a clear analysis and methodologies employed in the research implementation.

3.0.1 Nature of Data

Primary data was collected from two different sources. The first source was from two farms owned by one individual but wishes to remain anonymous. Data from the first source will be used to build and solve models 1, 2 and 3. The data include the crops that are grown, livestock, labour, water, fertilizer and capital employed in the two farms.

The other source of data was from Joe Tech Pvt and this data will be used to build model 4. The data includes prices of crops, forecast cost and probabilities of random events.

3.1 Model 1: Land Allocation Problem

3.1.1 Description

This model aims to find the optimal number of hectares that can be grown for each crop to maximize revenue from two farms. The model use linear programming to build and solve the problem. The variables for this problem, $x_1, x_2, \ldots, x_n$, represent the number of hectares of planted crop i.

3.1.2 Assumptions

1. Assume good weather and good harvest

2. Assume that all solutions and variables are always positive, for example, we cannot produce a negative output.

3. Capital constraint includes all input costs such as seeding, cultivation and pesticides.

4. The two farms operate separately and all resources needed for productivity are employed (full employment).

5. All costs and prices are pegged in USD.

6. Prices and costs do not change in the period of study.

3.1.3 Formulation

In this model, revenue (R) from sale of crops is maximized subject to land,labour, water, fertilizer and capital constraints and p is the price of crop i. The number of hectares $x_1, x_2 \ldots, x_n$ that can be grown are assumed to be greater than zero. The formulation of the problem is given by:

$$\text{Maximize } R(\mathbf{X}) = \mathbf{p}^{\top}\mathbf{X} \tag{3.1.1}$$

Subject to

$$\begin{aligned} \mathbf{AX} &\leq \mathbf{b} \\ \mathbf{X} &\geq \mathbf{0} \end{aligned} \tag{3.1.2}$$

where

$$\mathbf{X} = (x_1, x_2 \ldots, x_n)^\top$$
$$\mathbf{p} = (p_1, p_2 \ldots, p_n)\top$$
$$\mathbf{b} = (b_1, b_2 \ldots, b_n)^\top$$
$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$$

3.1.4 Expected Results

The model is expected to produce results that show the allocation of land to different crops which in turn maximizes revenue and the variables are supposed to be positive in order to be interpretable.

3.2 Model 2: Maximizing Reveue fom Livestock

3.2.1 Description

The model finds the number of livestock and type of livestock that can be sold to maximize revenue. This model also uses linear programming method to solve the problem. Variables represents the number of livestock of type j.

3.2.2 Formulation

In this model, the revenue from sale of livestock is maximized subject to land,labour, water and capital constraints and p is the price of the livestock i. The model can be formulated as:

$$\text{Maximize} \quad R(\mathbf{X}) = \mathbf{p}^\top \mathbf{X} \tag{3.2.1}$$

Subject to

$$\begin{aligned} \mathbf{AX} &\leq \mathbf{b} \\ \mathbf{X} &\geq \mathbf{0} \end{aligned} \tag{3.2.2}$$

where

$$\mathbf{X} = (x_1, x_2 \ldots, x_n)^\top$$
$$\mathbf{p} = (p_1, p_2 \ldots, p_n)^\top$$
$$\mathbf{b} = (b_1, b_2 \ldots, b_n)^\top$$
$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$$

3.2.3 Assumptions

1. Livestock only suffer natural mortality.

2. Livestock born fit in the allocated land. No need to allocated more land to new livestock

3. Prices and costs do not change in the period of study

3.2.4 Results

The model is expected to produce results that show the number of livestock and type that maximize revenue and the variables are supposed to be non negative in order to be interpretable.

3.3 Model 3: Transportation Problem

3.3.1 Description

This model aims to minimize the transportation cost from the source to destinations. The fuel transportation cost per trip needs to be estimated. Demand and supply also needs to be estimated.

3.3.2 Assumptions

1. The transportation problem is balanced.

2. The only transportation cost is the fuel expense and fuel expense from destination to source is not considered.

3. The trucks that supply the goats to markets do not breakdown.

3.3.3 Formulation

The model aims to minimize total transportation cost (TTC) subject to demand and supply constraints. Variables in the model represent the allocation to destinations at the least cost and variables should be non negative. Let s_i, d_i be the supply and demand in the destination i. Given m sources and n destinations, the problem can be formulated as a LP model:

$$\text{Minimize} \quad TTC(\mathbf{X}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{3.3.1}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq s_i, \quad (i = 1, 2, \ldots, n) \tag{3.3.2}$$

$$\sum_{i=1}^{m} x_{ij} \geq d_i, \quad (i = 1, 2, \ldots, m) \tag{3.3.3}$$

$$x_{ij} \geq 0, \quad \forall i, j$$

3.4 Model 4: Decision under Risk and Uncertainty

3.4.1 Description

This model aims to solve decision problem by farmers under a risk and uncertainty environment. The farmer wishes to find the crop to grow between various crops that gives the highest payoff also taking into account that the weather can be cold or warm and also that prices can go up or down. The model uses the decision tree analysis to find the crop that gives the best payoff.

3.4.2 Assumptions

1. Probabilities can be estimated.

2. Two choices or strategies are available ,but one of the choices is optimal.

3.4.3 Formulation/Building

Probabilities of random events are calculated and presented on a decision tree diagram. Backward induction is used in the decision tree in order to find the crop that gives the highest payoff. The basic algorithm steps are as follows:

1. The tree starts as a single node representing a decision has to be made.
2. Two branches from the node are formed. The farmer can either do a forecast of weather or do not forecast say.
3. If the farmer chooses to hire an expert to forecast the weather, branches are formed representing the forecaster can forecast a warm or cold year with a given probabilities.
4. Two branches are formed, if say the forecasting expert forecast a warm year, the farmer can grow either crop i or j.
5. Two branches are formed, if the farmer chooses to grow crop i, prices can go up or down with given probabilities
6. If the farmer chooses not to hire an expert to forecast the weather, the process is the same except that there is no forecasting branch.

3.4.4 Expected Results

The expected results of this model is to provide the farmer with a decision which is based on the highest payoff of a crop.

3.4.5 Software Used

The research utilized optimization tools to obtain results and achieve the primary objectives outlined in chapter 1. Python was used to optimize the linear programming problems (Model 1 and 2). Python was also used to solve the transportation problem. Excel was used to design a tree diagram. Matlab was used for data analysis and latex all chapters were presented using latex.

3.4.6 Conclusion

This chapter gave a general overview of the methods used in the literature review and how it was put into practice to produce the outcomes mentioned in the next chapter. This chapter described how the researcher addressed the data's initial issues before attempting into the model construction of the actual problem.

Chapter 4

Data Analysis and Results

This chapter depicts the findings and analysis of the data utilized in this investigation. The chapter aims to solve the farming problems of optimization, transportation problem and making decisions under risk and uncertainty. Data was observed and estimated from a farm owner in Norton but wishes to remain anonymous and another source was from JoeTech Pvt. In this chapter the models are described, formulated, build and results are presented.

Estimated Prices and Costs for Farms

The data from this section was observed and estimated from two farms in Norton. The farms have an estimated of 500 hectares of land,250 hectares for Farm 1 and the other 250 hectares for Farm 2. Prices and costs of farm activities were estimated with current market prices (2022 and 2023). The activities on the farms include crop growing and rearing of livestock which is then sold. Data from the two farms can be found in Table 1,2,3 and will be used to formulate and solve models 1, 2 and 3. The labour, water, fertilizer required and yield are per **hectare**.

Table 4.1: Observed and Estimated data

Action	**Farm 1**	**Farm 2**
Total available land(Crops)	250 hectares	250 hectares
Total available hours(Crops)	35000 man-days	35000 man-days
Total water available(m^3)(Crops)	3500000	3500000
Labour Cost should not exceed	$40000	$40000
Total available fertilizer(kg)(Crops)	1600000	1000000
Total available land(Livestock)	100 hectares	*
Total available hours(livestock)	1000 man-days	*
Total available water(livestock)	1000 (m^3)	*
Total available capital(livestock)	2000	*

Farm 1

Table 4.2: Estimated Prices and Costs for Crops

Type of Crop	**Labour Cost**	**Water needed (m^3)**	**Fertilizer needed (kg)**	**Selling Price ($)**
Tobacco	200	250	200	4300
Wheat	200	650	150	640
Potatoes	300	550	160	1600
Sweet Potatoes	200	500	2000	393
Beans	300	500	2000	1067

Table 4.3: Estimated Prices and Costs for Livestock

Livestock	**Labour Cost**	**Water needed (m^3)**	**Capital**	**Selling Price ($)**	**Available Hours**
Cattle	20	25L/day/cattle	2/day/cattle	$5000/10 Cattle	3hrs/day for 20 cattle
Pigs	10	9L/day/pig	1.1/day/pig$	600/10 Pigs	2hrs/day for 30 Pigs
Goats	15	5L/day/goat	0.8/day/goat	$300/10 Goats	1hr/day for 40 Goats
Layers	5	0.25L/day/layer	0.1/day/layer	$54000/10000 Layers	3hr/day for 90 Layers
Road runners	5	0.4L/day/Road runner	0.05/day/goat	$40000/10000 Roadrunners	1hr/day for 100 Road runners

Farm 2

Table 4.4: Estimated Prices and Costs for Crops

Type of Crop	Labour Cost	Water required (m^3)	Fertilizer (kg)	Selling Price ($)
Sorghum	100	800	100	248
Sunflower	300	1000	120	524
Soyabeans	100	800	160	451
Maize	400	479	200	248
Sesame	300	450	100	1185

Table 4.5: GMB Grain Price 2022

Commodity/Grain	Produce Price($USD/Tonne)
Wheat	640
Sunflower	524
Soya beans	451
Sugar Beans	1067
Sorghum	248
Sesame	118
Maize	248
Potatoes	1600
Ground nuts	393

TIMB Tobacco Price 2022

Tobacco Price $USD 4300

Interbank Rate used , 1: 632.77 (October 2022)

4.1 Model 1: Land Allocation Problem

4.1.1 Problem Identification

This model aims to find the optimal number of hectares that can be grown for each crop to maximize revenue from the two farms. There are ten crops that can be grown.

The land allocation phases are shown in Figure (4.1.1)

Figure 4.1: Land allocation phases

4.1.2 Problem Formulation

To formulate the Linear programming problem, let x_1, x_2, x_3, x_4, x_5 be the number of hectares of planted crops i in farm 1 and $x_6, x_7, x_8, x_9, x_{10}$ be the number of hectares of planted crops j in farm 2. The LPP becomes:

$$\begin{aligned}\text{Maximize Revenue} &= 4300x_1 + 640x_2 + 1600x_3 + 393x_4 + 1067x_5 \\ &\quad + 248x_6 + 524x_7 + 451x_8 + 248x_9 + 1185x_{10}\end{aligned}$$

Subject to
(Total land constraint)

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \leq 500$$

(Land Constraint Farm 1)

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 250$$

(Land Constraint Farm 2)

$$x_6 + x_7 + x_8 + x_9 + x_{10} \leq 250$$

(Total Labor Cost Constraint)

$$200x_1+200x_2+300x_3+200x_4+300x_5+100x_6+300x_7+100x_8+400x_9+300x_{10} \leq 80000$$

(Labor Cost Constraint Farm)

$$200x_1 + 300x_2 + 100x_3 + 100x_4 + 100x_5 \leq 40000$$

(Labor Cost Constraint Farm 2)

$$300x_6 + 200x_7 + 250x_8 + 360x_9 + 300x_{10} \leq 40000$$

(Total available Hours Constraint)

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \leq 70000$$

(Available Hours Farm 1 Constraint)

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 35000$$

(Available Hours Farm 2 Constraint)

$$x_6 + x_7 + x_8 + x_9 + x_{10} \leq 35000$$

(Total water Constraint)

$$250x_1+650x_2+550x_3+500x_4+500x_5+800x_6+1000x_7+800x_8+479x_9+450x_{10} \leq 7000000$$

(Water Farm 1 Constraint)

$$250x_1 + 650x_2 + 550x_3 + 500x_4 + 500x_5 \leq 3500000$$

(Water Farm 2 Constraint)

$$800x_6 + 1000x_7 + 800x_8 + 479x_9 + 450x_{10} \leq 3500000$$

(Total Fertilizer Constraint)

$$200x_1+150x_2+160x_3+2000x_4+2000x_5+100x_6+120x_7+160x_8+200x_9+100x_{10} \leq 2600000$$

(Fertilizer Farm 1 Constraint)

$$200x_1 + 150x_2 + 160x_3 + 2000x_4 + 2000x_5 \leq 1600000$$

(Fertilizer Farm 2 Constraint)

$$100x_6 + 120x_7 + 160x_8 + 200x_9 + 100x_{10} \leq 1000000$$

(Capital Constraint)

$$30x_1 + 20x_2 + 20x_3 + 10x_4 + 10x_5 + 10x_6 + 30x_7 + 20x_8 + 10x_9 + 30x_{10} \leq 200000$$

$$40 \leq x_1 \leq 50$$

$$25 \leq x_2 \leq 50$$

$$40 \leq x_3 \leq 50$$

$$5 \leq x_4 \leq 50$$

$$30 \leq x_5 \leq 50$$

$$30 \leq x_6 \leq 50$$

$$40 \leq x_7 \leq 50$$

$$10 \leq x_8 \leq 50$$

$$10 \leq x_9 \leq 50$$

$$25 \leq x_{10} \leq 50$$

Variables $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$ represents the number of hectares for tobacco, wheat, potatoes, sweet potatoes, beans, sorghum, sunflower, soyabeans, maize and sesame respectively

4.1.3 Results

The model produced results showing the number of hectares that needs to be allocated for each crop. Table 4.6 shows a crop production plan that maximizes revenue.

Table 4.6: Optimal land allocations and Revenue

Farm 1	**Land use/ha**	**Revenue/ha**	**Farm 2**	**Land use/ha**	**Revenue/ha**
Tobacco	50	215000	Sorghum	30	7440
Wheat	50	32000	Sunflower	50	26200
Potatoes	40.3	64480	Soyabeans	10	4510
Sweet Potatoes	5	1965	Maize	10	2480
Beans	30	32010	Sesame	49.67	58858.95
Total land use	**175.3**	**345455**	Total land use	**149.67**	**99488.95**

The LP results show that the farm manager should apportion 50 ha for tobacco, 50 ha for wheat, 40.3 ha for potatoes, 5 ha for sweet potatoes, 30 ha for beans in farm 1. In farm 2, apportion 30 ha for sorghum, 50 ha for sunflower, 10 for soyabeans, 10 ha for maize and 47.67 ha for sesame. The results show that more hectares should be apportioned to crops sold at higher prices such as tobacco, wheat and sunflower. The results also show that crops such as sweet potatoes sold at a lower price should be allocated a small land. In addition, although the farmer has 250 ha for farm 1 available to maximize revenue only 175.3 ha should be used. Similarly, only 149.67 ha out of 250 ha should be used in farm 2. The maximum revenue that can be expected with this crop production plan is $444993.33

4.2 Model 2: Maximizing Revenue from Livestock

4.2.1 Problem Identification

This model finds the number of livestock and type that should be sold to maximize revenue.

4.2.2 Formulation

To formulate the LPP , let $x_{11}, x_{12}, x_{13}, x_{14}, x_{15}$ be the number of livestock of type $1j$ where

$j = 1$ represents the number of cattle

$j = 2$ represents the number of pigs

$j = 3$ represents the number of goats

$j = 4$ represents the number of broiler chickens
$j = 5$ represents the number of road runners
The model is formulated as follows:

$$\text{Maximize Revenue} = 500x_{11} + 60x_{12} + 30x_{13} + 0.54x_{14} + 0.4x_{15}$$

Subject to

$$\begin{aligned}
0.404x_{11} + 0.0505x_{12} + 0.067x_{13} + 0.0404x_{14} + 0.0404x_{15} &\leq 100 \quad \text{(Land constraint)} \\
0.15x_{11} + 0.067x_{12} + 0.25x_{13} + 0.033x_{14} + 0.01x_{15} &\leq 1000 \quad \text{(Labour hours constraint} \\
25x_{11} + 9x_{12} + 5x_{13} + 0.25x_{14} + 0.4x_{15} &\leq 1000 \quad \text{(Water constraint)} \\
2x_{11} + 1.1x_{12} + 0.8x_{13} + 0.1x_{14} + 0.05x_{15} &\leq 2000 \quad \text{(Capital constraint)} \\
20 &\leq x_{11} \leq 40 \\
60 &\leq x_{12} \leq 100 \\
80 &\leq x_{13} \leq 100 \\
5000 &\leq x_{14} \leq 10000 \\
5000 &\leq x_{15} \leq 20000
\end{aligned}$$

4.2.3 Results

Table 4.7: Optimal number of livestock and Revenue

Livestock	**Number of livestock**	**Revenue**
Cattle	20	10000
Pigs	60	3600
Goats	80	2400
Broilers chicken	5000	2700
Road runners	15000	6000
Total Revenue		**20700**

The results show that the farmer can sell 20 cattle, 60 pigs, 80 goats, 5000 broiler chickens, and 15000 road runners in order to maximize revenue. The maximum revenue that can be achieved by selling livestock is \$20700.

The farmer has to combine the revenue from the two models. Total Revenue from farms= Model 1 Revenue + Model 2 Revenue = \$465693.33

4.3 Model 3: Transportation Problem in Farming

4.3.1 Problem Identification

The model aims to find the optimal allocation of goats to different towns at the minimum transportation cost. Farm 1 rears livestock in Norton and Ruwa. It supplies goats to restaurants with a large customer base in Damafalls, Hopely, CBD, Epworth, Tafara, Mabvuku and Kadoma. The farmer supplies goats at his own expense. Figure (4.2) describes the transportation problem from sources to desired destinations.

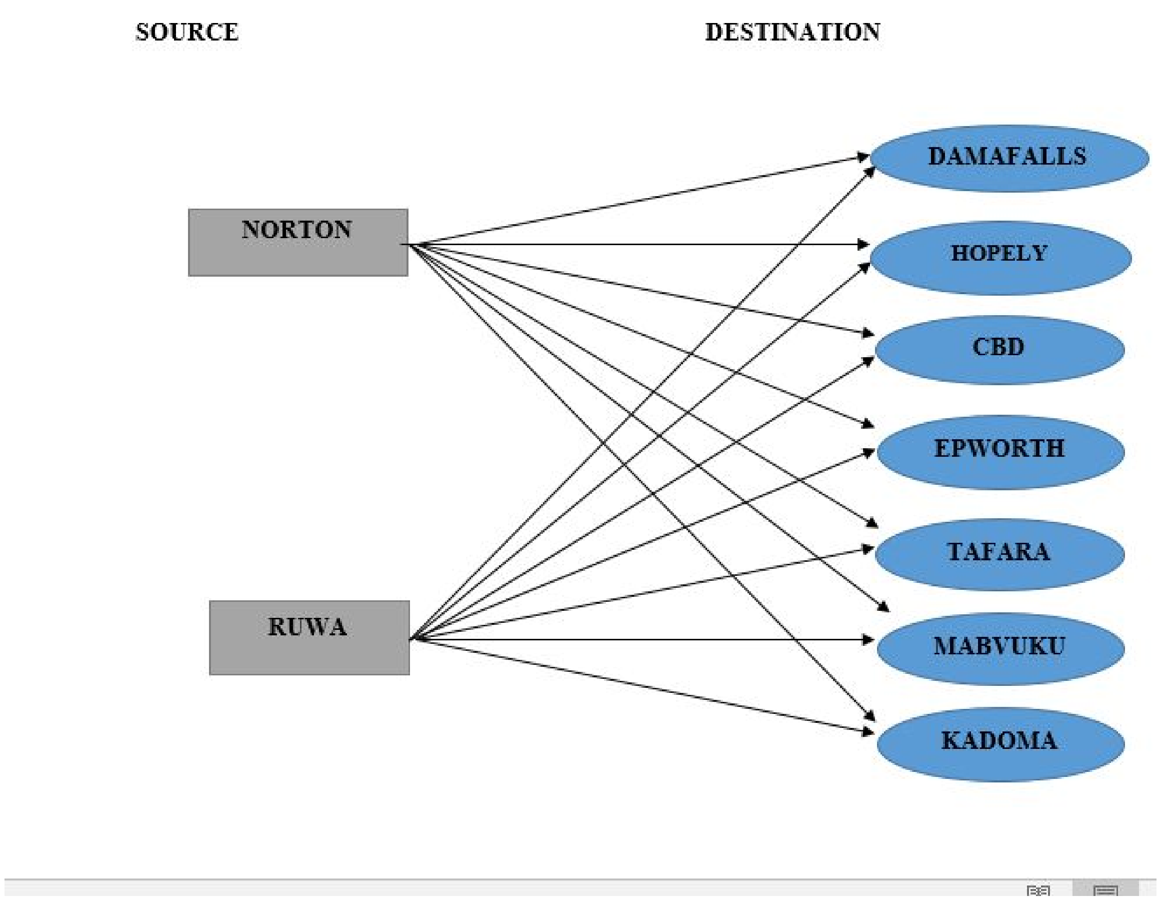


Figure 4.2: Transport Network Diagram

Formulation and Building

The model can be formulated as follows:

$$\begin{aligned}\text{Minimize} \quad z(\mathbf{X}) \;=\; & 7.7y_{11} + 4.95y_{12} + 4.62y_{13} + 6.71y_{14} + 7.403y_{15} + 7.04y_{16} + 11.77y_{17} \\ & + 1.1y_{18} + 4.125y_{19} + 2.31y_{20} + 1.1y_{21} + 1.32y_{22} + 1.518y_{23} + 18.238y_{24}\end{aligned} \tag{4.3.1}$$

Subject to

$$y_{11} + y_{12} + y_{13} + y_{14} + y_{15} + y_{16} + y_{17} \leq 150 \tag{4.3.2}$$
$$y_{18} + y_{19} + y_{20} + y_{21} + y_{22} + y_{23} + y_{24} \leq 150 \tag{4.3.3}$$
$$y_{11} + y_{18} \geq 50 \tag{4.3.4}$$
$$y_{12} + y_{19} \geq 20 \tag{4.3.5}$$
$$y_{13} + y_{20} \geq 20 \tag{4.3.6}$$
$$y_{14} + y_{21} \geq 60 \tag{4.3.7}$$
$$y_{15} + y_{22} \geq 50 \tag{4.3.8}$$
$$y_{16} + y_{23} \geq 50 \tag{4.3.9}$$
$$y_{17} + y_{24} \geq 50 \tag{4.3.10}$$
$$\forall y_{ij} \geq 0, \quad (i, j = 1, 2 \ldots)$$

4.3.2 Results

Table (4.8) and (4.9) shows the estimated distances between sources and destinations and the cost between sources and destinations.

Table 4.8: Distances Between Sources and Destinations

	Destination						
Town/ Source	**Damofalls**	**Hopely**	**CBD**	**Epworth**	**Tafara**	**Mabvuku**	**Kadoma**
Norton	70	45	42	61	67.3	64	107
Ruwa	10	37.5	21	10	12	13.8	165.8

Table 4.9: Transportation Cost per Goat, Quantity Demanded and Supplied

Destination								
Town	**Damofalls**	**Hopely**	**CBD**	**Epworth**	**Tafara**	**Mabvuku**	**Kadoma**	s_i
Norton	7.7	4.95	4.62	6.71	7.403	7.04	11.77	150
Ruwa	1.1	4.125	2.31	1.1	1.32	1.518	18.238	150
d_j	50	20	20	60	50	50	50	*

Data for the distances was calculated from the distance calculator https://www.distancecalculator.net

A truck Toyota Toyoace 2 ton vehicle with fuel consumption of 12.2km/L was used as the transporting vehicle and transports 10 goats per trip.
Fuel consumption per km when the truck is loaded is 1.5km/L
Cost of fuel=$1.65 (Zuva, March 2023)

$$\text{Total fuel consumption per trip} = \frac{\text{Total fuel consumption for the trip}}{\text{Fuel consumption per km when truck is loaded (1.5L/km)}}$$

$$\text{Total cost of fuel} = \text{Total fuel cost for trip} \times \text{Cost of fuel (\$1.65)}$$

$$\text{Fuel cost per goat} = \frac{\text{Total fuel Cost}}{10}$$

Figures (4.3), (4.4), (4.5), (4.6) give a summary of distances and costs between sources and destinations in bar graphs.

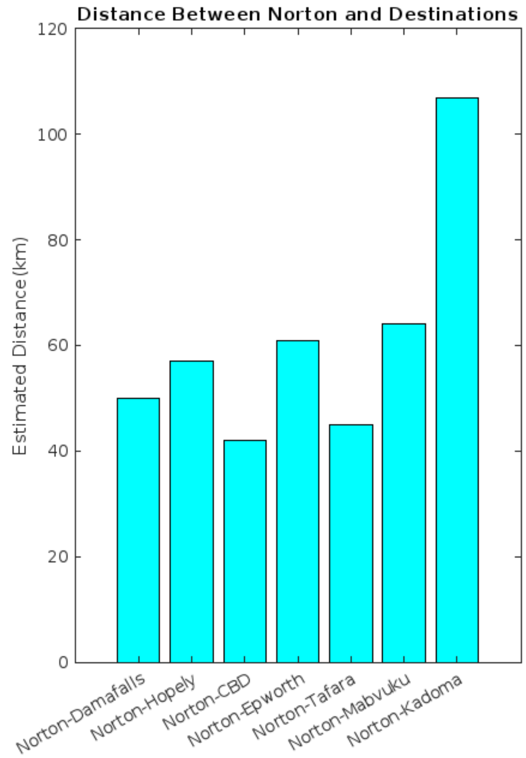


Figure 4.3: Distance between Norton and destination

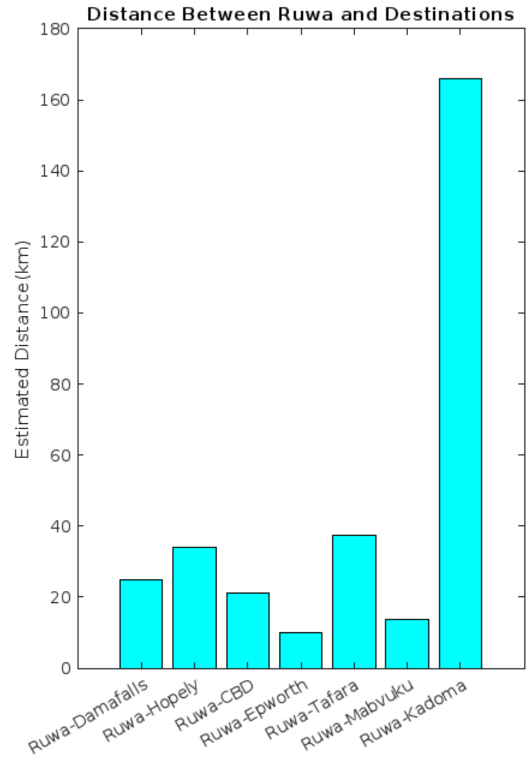


Figure 4.4: Distance between Ruwa and destination

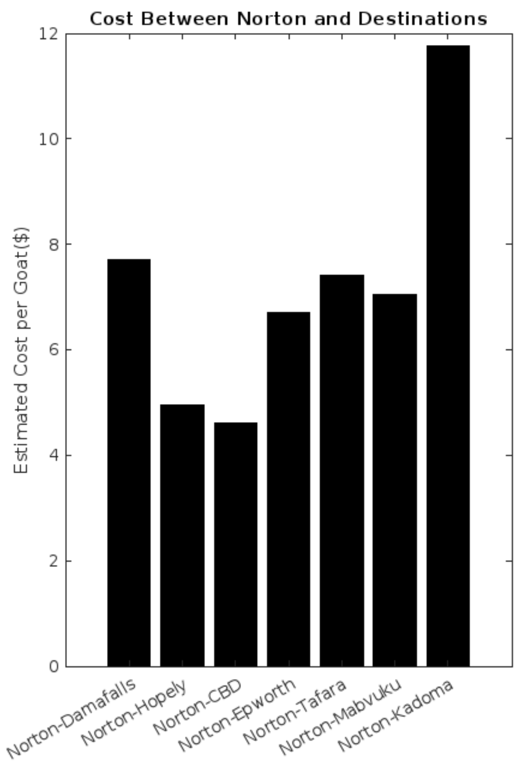


Figure 4.5: Cost between Norton and destination

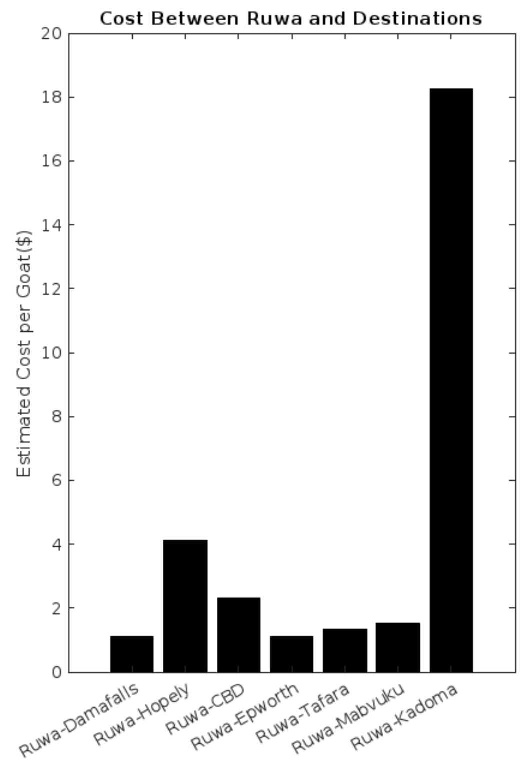


Figure 4.6: Cost between Ruwa and destination

Table 4.10: Optimal Solution Table

Name of Method	**Best Allocation**	**Optimal Solution**
Vogel Approximation	$y_{11} = 0.0$, $y_{12} = 20.0$ $y_{13} = 20.0$, $y_{14} = 10$, $y_{15} = 0.0$, $y_{16} = 50$, $y_{17} = 50$, $y_{18} = 50$ $y_{19} = 0.0$, $y_{20} = 0.0$ $y_{21} = 50$, $y_{22} = 50.0$ $y_{23} = 0.0$, $y_{24} = 0.0$	\$1375.0
Simplex	$y_{11} = 0.0$, $y_{12} = 20.0$ $y_{13} = 20.0$, $y_{14} = 10$, $y_{15} = 0.0$, $y_{16} = 50$ $y_{17} = 50$, $y_{18} = 50$ $y_{19} = 0.0$, $y_{20} = 0.0$ $y_{21} = 50$, $y_{22} = 50.0$ $y_{23} = 0.0$, $y_{24} = 0.0$	\$1375.0

Table 4.11: Optimal Allocations to Destinations

	Destination						
Town/ Source	**Damofalls**	**Hopely**	**CBD**	**Epworth**	**Tafara**	**Mabvuku**	**Kadoma**
Norton	0.0	20.0	20.0	10.0	0.0	50	50
Ruwa	50.0	0.0	0.0	50	50.0	0.0	0.0

In figures (4.5) and (4.6) the bar graphs show that long distances such as Norton to Damofalls, Norton to Kadoma and Ruwa to Kadoma have higher cost of transportation.The results from table (4.11) shows that from Norton the allocation of goats should be best suited to destinations such as Mabvuku and Kadoma and from Ruwa the best allocations to destinations should be to Damofalls, Epworth, and Tafara. In general, the model has allocated a large number of goats to areas where the transportation cost is at a minimum.

4.4 Model 4: Decisions under Risk and Uncertain

4.4.1 Problem Identification

This model aims to solve the decision problem of farmers in a risky and uncertain environment. The model makes use of a decision tree diagram and estimated probabilities. Data was collected from JoeTech Pvt in Harare.

JoeTech private company wants to venture into farming and has 50 hectares of land. It has to decide whether to grow maize or sunflower. If it grows one of the crops, the year could be cold or warm. The probability that the next harvest prices of the crops will up or go down is 0.8 and 0.2 respectively (We assume that prices will not be constant throughout the year in Zimbabwe). If the weather is warm and prices go up, maize and sunflower will give a payoff of \$50000 and \$70000 respectively. However, if the weather is warm and prices go down maize, sunflower and potatoes will give the farmer losses of \$20000 and \$15000 respectively. If the weather is cold and prices go up the payoff of maize, sunflower and potatoes are \$45000 and \$52000 respectively. However, if the weather is cold and prices go down, maize, sunflower, and potatoes will make the farmer suffer losses of \$35000, \$8000, and \$33000. Over the years, 50% of the years are warm and 50% have been cold. Before planning, Mr. Shumba can pay \$1000 to an expert to forecast the weather. If the year is cold, there is an 80% chance that the forecaster will predict a cold year. If the year is a warm year, there is a 70% chance the forecaster will predict a warm year.

Estimated Prices for Tobacco and Sunflower

Tables (4.12), (4.13), (4.14), (4.15), (4.16) and (4.17) provide data for estimated prices of the crops and estimated probabilities.

Table 4.12: Probabilities

Action	**Probability Price Up**	**Probability Price Down**
Grow Tobacco	0.8	0.2
Grow wheat	0.8	0.2

Year: 2022

Table 4.13: Prices for 2022

Action	**Prices/kg**
Grow Tobacco	\$4.30 (Zimbabwe Leaf Company, 2022)
Grow Sunflower	\$0.69 (GMB,2022)

Year: 2023

Table 4.14: Expected Prices in 2023

Action	**Expected Price/kg**	**Expected Price/kg**
If Prices Go	Up	down
Grow Tobacco	\$5	\$2.30
Grow Sunflower	\$0.7	\$3

Table 4.15: Payoffs

Action	**Price Up and Warm weather**	**Price Up and Cold weather**
Tobacco Payoff	\$50000	\$45000
Sunflower Payoff	\$70000	\$52000

Table 4.16: Output

Action	**Warm weather**	**Cold weather**
Tobacco Output	10000kgs	5000kgs
Sunflower Output	\$100000kgs	8000kgs

Table 4.17: Payoffs

Action	**Price Down and Cold weather**	**Price Down and Cold weather**
Tobacco Payoff	-\$20000	-\$35000
Sunflower Payoff	-\$15000	-\$8000

The company is also considering paying an expert to forecast the weather, whether it is going to be warm or cold on average throughout the year. The cost is \$1000.

4.4.2 Formulation

Let C represent that the weather is cold, W the weather is warm, FC represents the forecast predicts cold, FW the forecast predicts warm, U represents the prices go up and D the prices go down

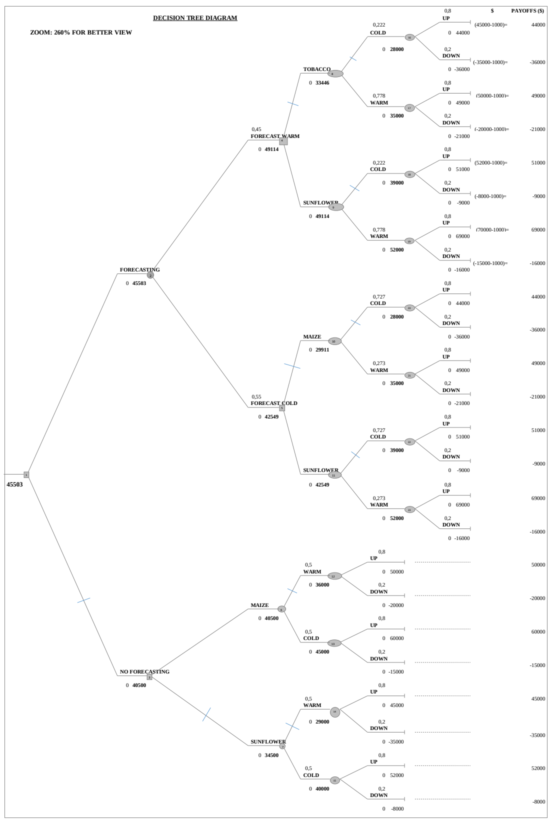
Then the probabilities are given below:

$$\begin{aligned} P(U) = 0.8 \quad & P(C) = 0.5 \\ P(D) = 0.2 \quad & P(W) = 0.5 \\ P(FW \mid W) = 0.7 \quad & P(FC \mid W) = 0.3 \\ P(FC \mid C) = 0.8 \quad & P(FW \mid C) = 0.2 \end{aligned}$$

$$
\begin{aligned}
P(C \mid FC) &= \frac{P(C \cap FC)}{P(FC)} && (4.4.1)\\
\text{But, } P(C \cap FC) &= P(FC \mid C)P(C) && (4.4.2)\\
&= 0.8 \times 0.5 = 0.4 && (4.4.3)\\
P(FC) &= P(FC \cap W) + P(FC \cap C) && (4.4.4)\\
&= P(FC \mid W)(P(W) + P(FC \mid C)P(C) && (4.4.5)\\
&= 0.3 \times 0.5 + 0.4 = 0.55 && (4.4.6)\\
\text{Then, } P(C \mid FC) &= \frac{0.4}{0.55} = 0.727 && (4.4.7)\\
P(W \mid FC) &= 0.273 && (4.4.8)\\
P(FW) &= 0.45 && (4.4.9)\\
\text{Similarly, } P(W \mid FW) &= 0.778 && (4.4.10)\\
P(C \mid FW) &= 0.222 && (4.4.11)
\end{aligned}
$$

4.4.3 Results

The results for the model can represented in the figure$* * *$ and the researcher used Excel(Tree Plan package) to design the tree diagram.


DECISION TREE DIAGRAM
ZOOM: 260% FOR BETTER VIEW
$
PAYOFFS ($)
45503
FORECASTING
0 45503
0,45
FORECAST WARM
0 49114
TOBACCO
0 33446
0,222
COLD
0 28000
0,8
UP
0 44000
(45000-1000)=
44000
0,2
DOWN
0 -36000
(-35000-1000)=
-36000
0,778
WARM
0 35000
0,8
UP
0 49000
(50000-1000)=
49000
0,2
DOWN
0 -21000
(-20000-1000)=
-21000
SUNFLOWER
0 49114
0,222
COLD
0 39000
0,8
UP
0 51000
(52000-1000)=
51000
0,2
DOWN
0 -9000
(-8000-1000)=
-9000
0,778
WARM
0 52000
0,8
UP
0 69000
(70000-1000)=
69000
0,2
DOWN
0 -16000
(-15000-1000)=
-16000
0,55
FORECAST COLD
0 42549
MAIZE
0 29911
0,727
COLD
0 28000
0,8
UP
0 44000
44000
0,2
DOWN
0 -36000
-36000
0,273
WARM
0 35000
0,8
UP
0 49000
49000
0,2
DOWN
0 -21000
-21000
SUNFLOWER
0 42549
0,727
COLD
0 39000
0,8
UP
0 51000
51000
0,2
DOWN
0 -9000
-9000
0,273
WARM
0 52000
0,8
UP
0 69000
69000
0,2
DOWN
0 -16000
-16000
NO FORECASTING
0 40500
MAIZE
0 40500
0,5
WARM
0 36000
0,8
UP
0 50000
50000
0,2
DOWN
0 -20000
-20000
0,5
COLD
0 45000
0,8
UP
0 60000
60000
0,2
DOWN
0 -15000
-15000
SUNFLOWER
0 34500
0,5
WARM
0 29000
0,8
UP
0 45000
45000
0,2
DOWN
0 -35000
-35000
0,5
COLD
0 40000
0,8
UP
0 52000
52000
0,2
DOWN
0 -8000
-8000

Calculation of payoffs at nodes

Node $= 0.8 \times (\$44000) + 0.2 \times (\$-36000) = \$28000$

Node $= 0.8 \times (\$49000) + 0.2 \times (\$-21000) = \$28000$

$\vdots$

Node 1 = Comparing \$45503 and \$40500 , we take the larger figure which is \$45503

Optimal Payoff Choice

The results from figure$* * **$ show that the company should consider paying \$1000 to the weather forecaster and should grow sunflower whether the forecaster forecast a warm or cold year. The company could expect a payoff of \$45503 if this option is chosen.

Chapter 5

Discussions, Conclusions and Recommendations

This chapter presents an overall conclusion of the research, it discusses the limitations, recommendations and future research opportunities in the area of resource allocation in agriculture.

5.1 Discussion

Preliminary data was acquired from a farm owner with two farms in Norton and some of the data was also collected from JoeTech Private company in Harare. At the farms, the researcher discovered that although the two farms were large, none of the farms was maximizing revenue to an expected level. The researcher concluded that the causes of less revenue generation could be that the farmer uses traditional methods of farming by just estimating a crop production plan and also that high operating costs were a result of considerably high transportation costs.

After analyzing the data collected, the researcher developed a linear programming model which aimed at maximizing total revenue from the two farms subject to resources constraints. The outcomes from this model showed that the farmer should apportion 50 ha for tobacco, 50 ha for wheat, 40.3 ha for potatoes, 5 ha for sweet potatoes, and 30 ha for beans in farm 1. In farm 2, apportion 30 ha for sorghum, 50 ha for sunflower, 10 for soyabeans, 10 ha for maize and 47.67 ha for sesame. The results clearly show that more hectares should be apportioned to crops sold at higher prices such as tobacco, wheat and sunflower. The results also show that crops such as sweet potatoes sold at a lower price should be allocated a small land. In addition, although the farmer has 250 ha for farm 1 available to maximize revenue only 175.3 ha should be used. Similarly, only 149.67 ha out of 250 ha should be used in farm 2. The maximum revenue that can be expected with this crop production plan is \$444993.33

The researcher also formulated another linear programming problem to maximize revenue by selling livestock. The result showed that the farmer can sell 20 cattle, 60 pigs, 80 goats, 5000 broiler chickens, and 15000 road runners in order to maximize revenue. The maximum revenue that can be achieved by selling livestock is $20700.

The transportation problem model gave the farmer an insight into where to allocate the goats at the least cost. The results showed that from Norton the allocation of goats should be best suited to destinations such as Mabvuku and Kadoma and from Ruwa the best allocations to destinations should be to Damofalls, Epworth, and Tafara. In general, the model has allocated a large number of goats to areas where the transportation cost is at a minimum.

After collecting data from JoeTech company, the researcher used dynamic programming to help to come up with a decision on whether to grow maize or sunflower. The results showed that the JoeTech company should consider paying for a weather forecaster if the forecaster predicts that 2023 will be a warm year then the company can grow sunflower and expect to gain $49114. If the forecaster predicts that the year will be cold, the company still needs to grow sunflower and would expect $42549 payoff.

5.2 Conclusion

Small-scale farmers are contribute largely to the growth and development of the Zimbabwean economy. These farmers must take farming as a business and use the available knowledge applied in this paper in order to maximize their returns and be able to make rational decisions when faced with risk and uncertainty.

5.3 Recommendations and Future research

Since small scale farmers are pivotal to the economy of Zimbabwe, instead of relying on traditional methods of farming, they should consider investing to get knowledge on how to maximize revenue, profits, efficiency and minimize costs. Other researches knowledge that small scale farmer should utilize are the feed mix problem, optimizing crops pattern, crop rotation plan, scheduling and irrigation water.

The following are ideas for further research based on the findings of this dissertation:

1. Modify and extending the research particularly the land allocation problem to include multiobjectives.

2. The transportation problem extended to include other costs such delay costs, tollgate costs and wages for drivers. The research can be further advanced to maximize delivery service to markets.

3. The decision tree can be extended using machine learning to take into account other factors that the farmer has no control such as diseases to crops, changes in the interest for borrowing fund and labour risk.

5.4 Limitations

The research has certain limitations, but they should not be interpreted as impairing its significance; rather, they should be viewed as providing room for future research along any gaps that the current study may leave. The researcher collected data from a farm owner and made assumptions about other data that was not readily available.

5.5 Delimitation

The research did not explore are problems in farming, however, the research only focused on four crucial problems in farming, namely the land allocation problem, transportation problem, maximization of revenue from selling livestock, and making decisions under risk and uncertainty. The research used pertinent mathematical concepts to increase the credibility and validity of the research project.

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Appendix A

Python Script

```python
#Simplex Method to Model 1
from scipy.optimize import linprog
c = [-4300, -640,-1600,-393,-1067, -248,-524,-451,-248,-1185]
A = [
[1,1,1,1,1,1,1,1,1,1],
[1,1,1,1,1,0,0,0,0,0],
[0,0,0,0,0,1,1,1,1,1],
[200,200,300,200,300,100,300,100,400,300],
[200,300,100,100,100,0,0,0,0,0],
[0,0,0,0,0,300,200,250,360,300],
[1,1,1,1,1,1,1,1,1,1],
[1,1,1,1,1,0,0,0,0,0],
[0,0,0,0,0,1,1,1,1,1],
[250,650,550,500,500,800,1000,800,479,450],
[250,650,550,500,500, 0, 0, 0, 0,0],
[0,0,0,0,0,800,1000,800,479,450],
[200,150,160,2000,2000,100,120,160,200,100],
[200,150,160,2000,2000,0,0,0,0,0],
[0,0,0,0,0,100,120,160,200,100],
[30, 20, 20, 10, 10,10, 30, 20, 10,30]
]
b = [500,250,250,80000,40000,40000,70000,35000,35000,7000000,3500000,3500000,2600000,1600000,1000000,200000]
bnd = [(40, 50),(25, 50), (40,50),(5, 50), (30, 50),(30, 50),(40, 50), (10, 50),(10,50),(25,50)]
res = linprog(c, A_ub=A, b_ub=b,bounds=bnd, method='simplex')
print(f' Optimal Solution: x1={res.x[0]},x2={res.x[1]}, x3={res.x[2]},x4={res.x[3]}, x5={res.x[4]}, x6={res.x[5]}, x7={res.x[6]},x8={res.x[7]}, x9={res.x[8]},x10={res.x[9]}')
print (f'Max Revenue=${-res.fun}')
```

```python
#Model 2: Maximizing Revenue from Livestock
from scipy.optimize import linprog
c = [-500, -60,-30,-0.54,-0.4]
A = [
[0.404,0.0505,0.067,0.0404,0.0404],
[0.15, 0.067, 0.25, 0.033, 0.01],
[25, 9, 5, 0.25, 0.4],
[2,1.1,0.8,0.1,0.05]
]
b = [100,1000,1000,2000]
bnd= [(20,40),(60,100),(80,100),(5000,10000),(5000,20000)]
res = linprog(c, A_ub=A, b_ub=b, bounds=bnd, method='revised simplex')
print(f' Optimal Solution: x11={res.x[0]}, x12={res.x[1]}, x13={res.x[2]}, x14={res.x[3]} x15={res.x[4]}')
print (f'Max Profit=${-res.fun}')
```

```python
#MODEL 3: TRANSPORTATION PROBLEM
from pulp import *

inf = float('inf')

prob = LpProblem('problem', LpMinimize)

# INSTANCE DEFINITION
D= ["Damofalls","Hopely ","CBD","Epworth","Tafara","Mabvuku"," Chitungwiza"," Mbare"," Kwekwe"," Kadoma"]
S= [" Norton"," Ruwa"," Goromonzi"]

A = {" Norton":150," Ruwa":150," Goromonzi":150}
B = {"Damofalls":100,"Hopely ":90,"CBD":10,"Epworth":50,"Tafara":50,"Mabvuku":54," Chitungwiza":51," Mbare":20," Kwekwe":10," Kadoma":15}
cost_values = [
[7.7,4.95,4.62,6.71,7.403,7.04,6.27,5.5,19.36,11.77],#Norton
[1.1,4.125,2.31,1.1,1.32,1.518,3.7356,2.75,26.07,18.238],#Ruwa
[2.42,4.4,4.653,3.63,2.981,3.41,5.819,5.687,28.017,20.24]# Goromonzi
]

# DECISION VARIABLE GENERATION
C = {site:{company:cost_values[i][j] for j,company in enumerate(D)} for i,site in enumerate(S)}

E = [(i,j) for i in S for j in D if C[i][j] < inf]

x = LpVariable.dicts('x', E, lowBound=0)

# PROBLEM FORMULATION
prob += lpSum([C[i][j]*x[i,j] for (i,j) in E])

for i in S:
    prob += lpSum([x[i,j] for j in D if (i,j) in E]) == A[i]

for j in D:
    prob += lpSum([x[i,j] for i in S if (i,j) in E]) == B[j]

# SOLUTION
status = prob.solve()

print(f'STATUS\n{LpStatus[status]}\n')

print('SOLUTION:')
for v in prob.variables():
    print(f'\t\t{v.name} = {v.varValue}')

print('\n') # Prints a blank line
print(f'Minimum Total Cost: {prob.objective.value()}')
```